(AtomConcept) An Atomic Cognitive Architecture

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Abstract. The paper proposes an atomic concept, which is an abstract conceptual space. Atomicity means differentiable fundamental constituents that may compose to construct formality as much as ideality, which implies that (Form $\equiv$ Semantics, Word $\equiv$ Sentence, and Self $\equiv$ Word), and consequently, the bi-directional convertibility between the sensed analog world and the conceptual domain for constructing self-evolving agents. The atomic concept builds on a three-perspectival ontology of urbanism (a theory of meaning) that standardizes a dual representation of the atomic model, a differentiable manifold (local conceptual simulation) and a binary hierarchical graph (long-term memory recording the local simulations). The paper proposes seven abstract symmetric dimensions for the atomic manifold, which are distilled from the natural language's adpositions. The abstract dimensions are materialized by a proposed chamber geometry, an algebraic-geometric structure, to represent states (differentiable atomic 3D propositions) over any point of the manifold. These states are differentially changeable over the manifold by the means of parameterizable verb (force) actions, and that is how state-change of event structure is modelable over hierarchically generalizable $IF - THEN$ statements. The proposed 3D scene propositions are an abstract differentiable variant of formal first/higher-order logic languages. The paper proposes a recursive, invariant, hierarchically explainable Atomic Neural Network (AtomNN) as a materialization of the atomic graph/manifold duality, and it, accordingly, reconciles the symbolic and connectionist approaches. The AtomNN is populated by chamber geometric atomic propositions, and consequently, it is intrinsically algebraic-geometric multimodal. Similarity is assessed by algebraic homomorphically and geometric differentially comparability, and hence, learnable patterns are explainable. The paper promises the constructability of explainable AI, artificial self-aware agency, controllable AI, biological-like backpropagation (white box hierarchical) for training the atomic model, and, in general, AGI.

Keywords: abstract atomic conceptual space, computational cognitive geometry, cognitive linguistics, group theory for information theory

1 Introduction

Symbolism and connectionism are believed to be two pillars of cognitive processes. In symbolism, predicated entities are joined by logical connectors to communicate knowledge, e.g., the statement "if it is raining and I'm with John, then we will finish the research", maybe predicated and formalized as "$raining(today) \wedge with(I, John) \Rightarrow finish(we, the\,research)$", and the truthfulness of such a proposition may be evaluated against set-based models of joined predicates. Such symbolic representation is praised for its communicative openness and its relatedness to how human agents tend to cognize and deliberate specific entities as symbols. This approach is practically utilized by manually populating rules, as templates, hoping that formal manipulation of the symbolic statements may construct deductive systems. Nonetheless, the lack of proper differentiable/continuous modeling of the meaning of the predication and the logic operators led to the failure of this approach in areas like learnability, knowledge transferability, and the inability to model the analog perceptual/sensorimotor systems. On the other hand, connectionism may readily make sense of patterns in prepared training data, e.g. $(\dot{X}, Y)$, using differentiable/trainable connectionist models $M(X, \Theta) \rightarrow \hat{Y}$. The model's parameters Θ are optimized to reduce and generalize the cost of wrong predictions between $\hat{Y}$ and the training Y against the fixed predefined features X. The differentiability, and hence the trainability, of connectionist models places it at the forefront of contemporary AI without any imaginable future of the field without its contribution. Nonetheless, Connectionist neural models need a large amount of data to train the black-boxed parameters Θ of the predefined model. The training expenses, the fixed, specially-engineered input and output spaces X, $\hat{Y}$, and their black-box unexplainability made these models brittle to develop, use, and maintain.

In summary, the symbolism's lack of *differentiable modeling* of its constituting elements, and the connectionism's need for modeling *composable fundamental constituents* made both of these approaches unexplainable. Therefore, a single model that reconciles these two approaches and complements their shortcomings with their mutual strengths may prove to be the best path for constructing evolving artificial agents that are capable of auto-modeling the world and communicating over these constructed models. *"Meaning"* proves to be a unifying ground of the multimodal perceptual systems, as much as of the higher cognitive processes, and therefore, meaning is foundational for defining such a unifying model.

The paper proposes an abstract atomic concept that maintains the essence of *meaning*, which leads to standardizing ***state*** representation and the ***processes*** of changing these states. The ***state-change*** abstraction is materialized by a specifically defined geometric-algebraic structure, named chamber geometry, that sustains auto-defined typologies and their comparability. The proposed model $\mathcal{M}$ yields a contentious space of possibility, $\mathcal{M}(x) \implies M_{p \in P}$ (as a point p of the points P of a differentiable manifold M). The model $\mathcal{M}$ may model any x, as a continuous phenomenon, that includes the constituents of first/higher-order logic, e.g., $\{\vee, \wedge, \neg\}$, but with continuous implication spaces of $\{\implies, \iff\}$, rather than the limited $\{T, F\}$ of formal semantics. Additionally, the atomic model's propositions are abstractions of 3d scenes, and they may be quantified with the regular quantifiers $\{\forall, \exists\}$. But what is *meaning*? And, how does the paper proceed from that point on?

The paper adopts a three-perspectival ontology of urbanism [1; 2] to elucidate "what is meaning?". The ontology states that three contrastive perspectives, labeled rational (structural), emotional (systemic), and visual, are sufficient to explicate all the variants of any material or conceptual entity. The rational perspective dictates structurality, containment, boundedness, and true/false simplifications of hierarchical spatiality. The systemic perspective dictates interactivity between attracting/repulsive forces. The visual perspective is about describing these two perspectives by a conscious agent. We may emphasize the centrality of the three perspectives in dictating meaning over the following axiom, which is the first of only two axioms of the paper.

Axiom 1 (the three-perspectival sufficiency) *the three perspectives, labeled rational, systemic, and visual, may sufficiently explicate all the variants of any material or immaterial entity, which may collectively be named as a concept,which is labeled* $Con(label)$*.*

Both the rational and systemic perspectives are deemed objective, while the visual perspective is the subjective perspective responsible for depicting and materializing the other two objective perspectives. For example, no two sane individuals would disagree on the structure or action of an opening door or a moving vehicle, but describing that as fast or slow, near or far, suitable or not, good or bad, useful or useless is a practice of subjectivity.

The three perspectives are meant to bridge language and cognition. Applying this first axiom for modeling the meaning of a word of natural language is a good experiment of the axiom, and it may open a novel insight into open-word classes of natural language. For a word to mean, it has to maintain a version of the three perspectives. Meaning that it has to maintain structure, interactivity over these structures, and the descriptive materialization of structurality and interactivity, which develops by an experiencing agent. Consequently, the word is a memory device, which the paper presumes to be differentiable for learnability. A word in language is a concept in cognition, and such a minimalistic differentiable memory device is what the paper presumes to be the atomic concept. Therefore, for a concept $Con(label)$, label is a noun representing the name of the concept, and the concept maintains the other open word classes of interactivity (verbs), spatial structurality, and describing experiences (adjectives and adverbs). This means that the open word classes are just variant manifestations of a word (concept), and that is why linguists celebrate the dynamic and evolutionary aspects of open word classes. For example, an antenna may become "to antennaize" or antenna-like, etc., because all of the open word classes are entwined to create a concept. Composing concepts retrieves the proper memory about the concept into attention, and the following lemma states such conceptual relatedness.

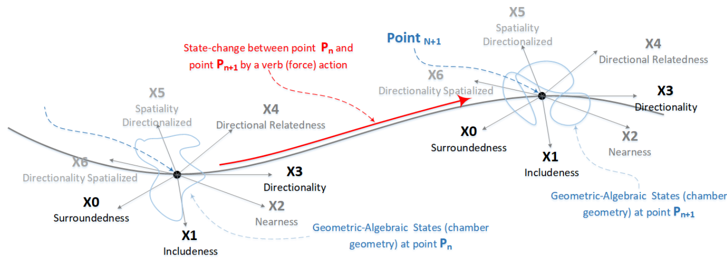

Fig. 1. The proposed mathematical modeling of *state-change* over the atomic manifold. The seven dimensions [x0-x6] are used to define the geometric-algebraic chamber geometry for modeling the *state-change* on several frames (points) on the diff. manifold.

Lemma 1 (relativistic semantics). *any concept develops meaning by relating to other concepts, either objectively over a spatial or interactive relatedness or subjectively over the perceiving points of view.*

We would now lay the second, and last, axiom of the paper that presumes a symmetric world. But before doing that, we need to explicate the constituents of the atomic concept. Although the three perspectives recognize the open word classes as various manifestations of a concept, the adpositions' closed word class (prepositions and postpositions) are presumed to construct the open classes' variants of structurality and interactivity. The other closed word classes are either connectors, e.g. the joining operators $\{\vee, \wedge, \neg, \implies, \iff\}$, which may act as conjunctions, while pronouns are just concepts (words), e.g. the concepts of "I", "We", "They", or "It" are mere memory devices of structures, interactions, and the experiences of describing them. In fact, the closed word classes are fixed, barely change, and are universal amongst all the known linguistic instances.

Figure (1) illustrates the constituents of the atomic concept. The atomic concept is a model of a differentiable memory device. The paper distills the natural language's adpositions into seven adposition dimensions used to structure both spatiality (structurality), interactivity (directionality), describability, swarm interactivity, and geometric analysis. The seven adposition dimensions are objectively defined, and over them, geometric instances are populated. These geometric instances are defined by the paper as chamber geometry that encodes spatial experiences and substantiates comparability. The chamber geometry encodes the algebraic-geometric spatial relationships in the form of 3d spatial propositions, joined by the smooth connectors of $\{\vee, \wedge, \neg\}$. These atomic propositions depict ***states*** , as abstract spatial relatedness, which may smoothly ***change*** , by the means of a force (verb) effect, and such changing shapes the differentiable atomic manifold as a differential memory device of a concept. Distilling natural language to depict states on a single frame, and then tracing the 3d proposition changeabilities defines the invisible affecting forces. The capacity to model state-change is not limited to the event structure of actor-action-patient but may include any entity, such as utilized instruments, environmental expectations, temporal conditionality, goals, purpose, etc. Nonetheless, all these constituents come in duals, and based on that, the following final axiom of the paper is laid:

Axiom 2 (the symmetric world) *meaning manifests in a symmetric relatedness. Any adposition dimension is represented as a group $\mathcal{G}$ containing opposite adpositions. For any descriptive geometry, there is a parametrized opposite polar* $\in [-1, +1]$. *And finally, for any action "a", there is an undoing anti-action "a^{-1}".*

A group is the mathematical tool for modeling symmetry, and that is how the adposition dimensions are modeled. All the proposed operators come in three material, behavioral, and inferential variants, and each is dually defined for the two inverse polars of the descriptive space. These connectives structure atomic 3D propositions that are hierarchically defined, from simplest types to more detailed and complex ones, using the generalizable $if - then$ statements. The atomic 3D propositions signify materializable states that may be used as arguments of a verb, which is represented in the paper as the $SE(3)$ group, and the six rotational and translational parameters of the $SE(3)$ verb parametrize the arguments' states. The infinitesimal changes in the verb parameters, the behavioral aspect, reflect changes on the arguments' states, material aspects, and the speed of such changeability is the third inferential state. Such infinitesimal dynamics are algebraically-geometrically simulated on local points of the atomic manifold and hierarchically recorded in the atomic graph (the long-term memory). Although the memory graph is populated by the subjectively embodied experiences (observed infinitesimal changeability), the subjective properties of the embodied agents may be identified. Therefore, communication between differently scaled agents, due to the symmetric group modeling, is still objectively defined. For example, a fly-sized robot and a jet-sized robot may unambiguously communicate their perceived and conceived processes using the atomic modeling. The following lemma states that invariance of content.

Lemma 2 (the invariance of meaning). *the meaning content of the structural configurations and the behavioral interactivity is invariant to the different viewpoints' materialization.*

Related Works

Conceptual spaces: Conceptual spaces are proposed by Gaerdenfors [3; 4] to unify the symbolic and sub-symbolic representations. It proposes fundamental dimensions to represent the perceptual realm, over which convex geometric spaces are laid to represent properties, domains, and concepts. Conceptual spaces have promising applications [5; 6], and they are deliberated by Lieto et al. [7] to unify cognitive architectures. Nonetheless, the proposed *concrete* dimensions are manually defined to populate physical perceptual phenomena, such as colors, and as such, they are unexplainable and are restricted to modeling perceptual phenomena. The paper closes this gap by proposing *group-based, contrastive, abstract* dimensions of a *smooth manifold*. The proposed seven adposition (preposition and postposition) dimensions distill and consume the known adpositions of natural language. Similar to conceptual spaces, the proposed chamber geometry uses convex 3d balls and vector dot products for comparability, and, hence, for classification, pattern recognition, and learning. Additionally, conceptual spaces may model event structures, but the atomic concept models conceptual propositions, which are algebraic-geometric abstractions of 3d scenes, and this enables the atomic concept of modeling *state-change* of the materialized perception and the immaterial ideal all alike, and in encoding (evolved learning) spatial experience.

Adpositions, language, and cognition: The role of adpositions in explicating semantics has been strongly debated by major linguists, cognitive linguists, and geometric cognitivists [8; 9; 10; 11; 12; 13]. For example, Talmy and Langacker [8; 9; 10; 11] associate semantics with the closed-word classes that lay the semantics structure, while open-word classes may maintain their meaning by their interrelated networking over the numbered structures offered by adpositions, which are used in modeling event structures. Similar to the paper's orientation, adpositions have been proposed by Chilton to equally represent the material (modeling) and the ideal (metaphorizing) by the Spatial-temporal structures offered by adpositions [12]. Nevertheless, the semantics of adpositions and verbal behaviors are laid out using concrete geometric examples, making them only suitable for communicating theorizations and ideas but not for knowledge transferability (generalization and analogical/similarity matching). The proposed atomic model surpasses this concreteness limitations by offering abstract topological seven dimensions, over which the differential modeling of chamber geometry is trainable, which opens the door for both algebraic homomorphic and differential scalar comparability.

Machine learning and AI: LLMs are at the forefront of revolutionizing contemporary AI. LLM transforms the fixed vector embedding of a predefined set of tokens. LLMs maintain attentive mechanics for context-based transformations. The LLM's inputs, outputs, and latent spaces are embedded in Euclidean spaces, which implies that measuring distances between different vector representations is the way to express similarity versus polarity. Such distance measurement reflects the highest level of abstraction that misses the wealth of content (meaning-based) expressiveness that may be otherwise available by the atomic concept's seven adposition dimensions and the comparable differentiable chambers geometry. Additionally, the black-boxed vector embedding and the trained weights of the transformation matrices imply unjustifiable and unexplainable modeling, which is presumed to be compensated by the atomic concept. Furthermore, the memory-associative (auto-completion) architectures, such as Hopfield networks and Boltzmann machines, minimize the energy function for the network to converge to a local minimum, as differential attractors of memorized Patterns of states of a trained manifold. Nevertheless, patterns themselves are not explainable, and the atomic model promises to justify the trainable parameters of any pattern that is trainable over the atomic manifold.

Paper Structure The atomic concept is a mathematical model that recognizes the symmetry found in the natural language's adoptions and distills such symmetry in seven dimensions used to model the structure and interactivity of a smooth manifold. The differential atomic manifold is populated by 3d scene propositions, constructing what the paper names chamber geometry. Based on that, the paper defines the mathematical model and proposes it as a 3d scene abstract language that explicates lingual/cognitive variations of any concept. And accordingly, the paper first introduces the model as a graph, in section (2), to outline the state-change memorized cognitions. In section (3), the bulk of the paper is laid out to describe the atomic manifold of the proposed descriptive chamber geometry. In section (4), learnability is deliberated, before closing the article with the discussion and conclusion sections.

2 The Atomic Model as a Graph

One of the influential contributions of the paper is to dually represent the atomic concept by a graph and a differentiable manifold. That is because meaning unifies the graph representation and the differential manifold, which leads to ***dually*** manifesting a *differentiable graph* as much as a networked *discretization of the Manifold*. But what is the benefit of such dualistic representation? Memory stores a large number of atomic propositions (3D scene or inference propositions). The propositions may be coming from different sources, e.g., sensed analog materials of 1D/2D manifolds (sounds and images), experienced 3D manifolds, or atomic-parsing of lingual statements, but they all encode the same state-change content represented by the atomic propositions. Retrieving suitable propositions for further processing in any given context is a crucial process that may otherwise render the memory system useless. The atomic graph supports global top-down searching while the atomic manifold supports local bottom-up navigation, and both of these searches complement each other for optimal local/global searching. Additionally, the atomic graph may fulfill the long-term memory needs, while the atomic manifold may fulfill the simulation needs of other memory types, e.g., working memory and procedural memory. Finally, merging the two approaches would fulfill the needs of the other known memory types.

The atomic graph is a hypertree, a variant of the tree graph. It records the hierarchical $if - then$ statements along with the speed of transition of one statement to another based on the related verb parameterization. The graph $G = \{V, E\}$ is defined by the set of vertices V connected with the set of edges E. The atomic graph maintains two versions of vertices, V_{sim} for local simulation of verbs (stored submanifolds of the atomic manifold), and V_{if} as labeled sets of $if - then$ atomic propositions, and both are named nodes. Each V_{if} node is labeled by the condition of the $if(condition_i)$ and the node contains the conclusions of this conditionality $then(conclusions_i)$. Both the conditions and the conclusions are atomic 3D propositions. Tree graphs are free of cyclic connections, and the atomic hypertree is the same. The $if(condition_0)$ starts a path of the atomic graph, where $condition_0$ is the simplest type that starts the path, and all consequent nodes on that path extend the complexity of the constituents of the prior conditional statements.

The atomic graph may be visualized as patches of *atomic neural networks*, see Section (4.3). It maintains the capacity of storing several manifolds, e.g., two atomic manifolds, one to model changeability and another for modeling the attentive system. The attentive manifold may model the speed of the flow of thought associated with the atomic manifold. These related speeds are encoded on the edges of the atomic graph. In the memory management section, see Section (4.4), after covering the atomic manifold and the atomic propositions, the node and edge types are represented in more detail.

3 Defining the Chamber Geometry (the smooth descriptive comparative geometry)

It is named "chamber geometry" because it is considered a materialization of the mathematical sets. In the heart of the chamber geometry is the *hierarchical type system* that substantiates the generalized materialization of *set belonging* that suits the needs of perceptual and cognitive systems alike. The *conditions* of set belonging and the *features* of entities belonging to the materialized set are defined by a proposed $if - then$ operator. The atomic type system builds on seven symmetric dimensions and their cross-products that are distilled from lingual adpositions. These seven dimensions characterize smooth manifolds representing position ($SE(3)$ group), directionality ($SE(3)$ group), scalar polarized descriptive bounded domains $\gamma = [-1, +1]$, and bounded variants of vector field segmentation or spatial solids generating vector fields. These are believed to be the bare minimum needed to model any system, let that be material, immaterial, or modeling the thinking process itself (meta reasoning). These basic dimensions may be mathematically joined by different operators $\{\vee, \wedge, \neg, \implies, \iff\}$ to construct the atomic propositions (3D scene manifold) that may dually model constrained and allowed spaces.

The atomic concept is an interdisciplinary model that utilizes minimalistic notions to unify what otherwise are considered disparate fields. The consequence of such interdisciplinarity is the use of special terminology adopted for specific intentions, and to avoid disorientation the paper rigorously defines any specially used terminology when needed.

Section structure: The section starts by introducing the mathematical abstractions of the seven dimensions and their cross-products, which collectively are named "adposition dimensions". Following that the chamber geometry is defined as instantiated materializations defined over the abstract adposition dimensions. Joining these instantiated geometries by the connectives $\{\vee, \wedge, \neg, \implies, \iff\}$ constructs the 3D scene propositions, and building on that the type system is defined along with its hierarchical, comparative, and convertible characteristics. The deductive system is explicated as a *geometrical* and *algebraic* necessitated implications of any 3D proposition. Finally, the $if - then$ operator is defined before the section closes by a representation of the atomic manifold that is synchronized with the proposed atomic graph's representation.

The reader may skim this section during the first reading by gaining a basic understanding of the *three versions* of the connective operators, atomic hierarchicality, type convertibility, the nesting $if - then$ statement, and the type system, as a prepartion for the next learnability section, after which, the intricacies of these operators may be more appreciated.

Smooth and finite groups are used extensively throughout the paper. A mathematical group is a fundamental structure in abstract algebra that captures the idea of symmetry, transformations, and operations that can be reversed. A group G is a set equipped with two operators $\{\cdot, -1\}$, and a neutral element (the identity e). The two operators are closed (there is always an element of G satisfying any formula) and associative (functional compositionality). Any group may act on a set (G-set) or a model (G-model) by permuting its elements and partitioning the set elements into orbital equivalence classes. In a smooth group, the two operators are smooth as well.

3.1 Defining the Adposition Dimensions

Adpositions are a main class of closed word classes. Its foundational contribution to structuring semantics equally spans all known natural language instances, and cognitive linguistics recognizes their role in structuring both higher cognitive and perceptual systems alike. Consequently, the paper proposal for utilizing adpositions as dimensions of the cognitive manifold may be supported by a wealth of neurological, cognitive, and linguistic evidence. Nonetheless, using adpositions as dimensions is challenging because adpositions are rich in their polysemic content. For example, the prepositions "Over" is investigated to signify more than forty-one meanings. The paper resolves such ambiguity by stripping adpositions down to pure mathematical representations of the semantic quanta offered by the three perspectives. The combination of these fundamental mathematical representations constructs adposition phenomena that unambiguously represent any specific sense of any adposition of any natural language.

The paper distills the adpositions into seven abstract representations. The seven dimensions are believed to suffice the needs of any modeling scenario, as would later be used to construct rich variants of the atomic propositions. The seven dimensions are symmetric and they grasp the notions of:

- X_0 Surroundedness: it models position in a relative space. Semantically, relative objects are located on the surface of a ball.
- X_1 Includedness: it models causality and reasoning, it is a materialization of the conditional set belonging, and it models the hierarchically nesting $if - then$ propositions.

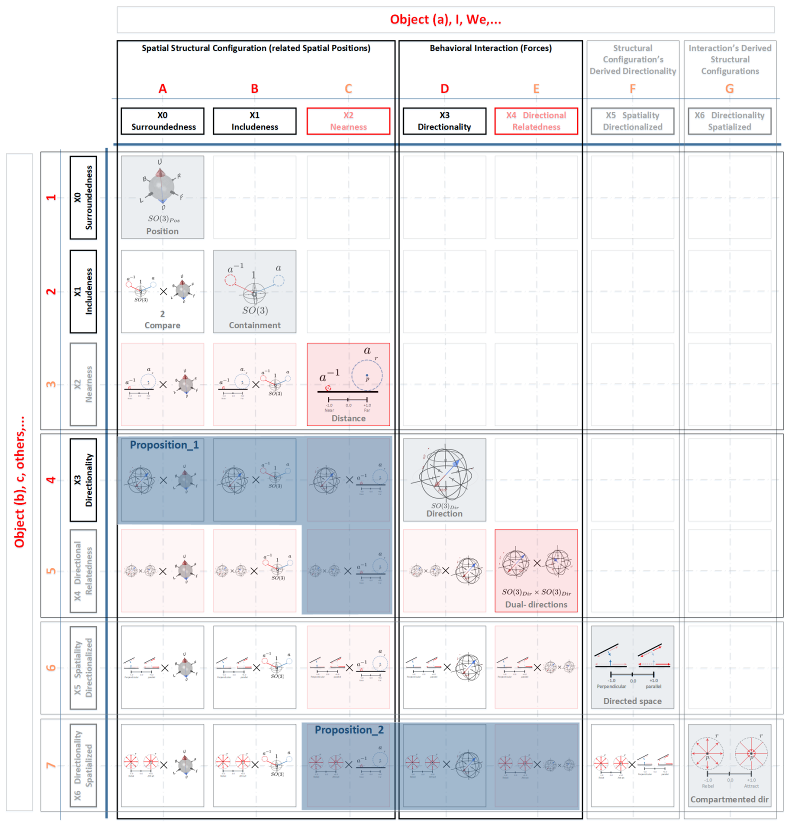

Fig. 2. different exemplar geometric instantiations of the types defined over the adposition dimensions

- X_2 Nearness: it models relative comparability between concepts. Learnability builds on the comparability of this dimension.
- X_3 Directionality: it models relative directions in space as vectors pointing from the origin of a ball to a point on its surface.
- X_4 Directional relatedness (modeling swarms): it models relative relatedness between different directions.
- X_5 Spatiality Directionalized: both X_5 and X_6 are meant to analyze geometric entities, e.g., curves and surfaces, and vector fields as part of vision streams or geometric analytical purposes. X_5 approximates curves or surfaces using different-sized balls, $B(r, p)$, that may be mathematically modeled using Fourier transforms, signal domain, which may be a variant of circle-approximated curves. As a result of X_5 analysis, geometry-generated parallel/normal vector fields are derivable.

– X_6 Directionality spatialized: it segments vector fields into rebelling/contracting regions using complementary processes to X_5

These seven variations are deemed fundamental for modelling any phenomenon. In addition to these seven dimensions, their respective cross-products, e.g., $X_0 \times X_4$, define types by their own. For example, colors, temperature, speed, swarms, etc., are represented as fundamental native types (inseparable cross products). These types are the basis of the paper's proposed hierarchical type system (see Figure 2). Any atomic proposition, no matter how complex it may be, is convertible to any of the seven adposition dimensions or their cross products.

The Abstract Mathematical Representations: All seven dimensions represent symmetry between two related entities. For example, if a is at the south of b, then b is at the north of a, and the same applies for the rest of the dimensional typologies. The mathematical structure that models any form of symmetry is known as a group. The paper employs $SO(3)$, the special orthogonal group, and the cyclic group C_3, acting on a $SO(3) \times SO(3) \times SO(3)$ space, to model positional, directional, and inclusion dimensions. Nonetheless, specially defined groups are needed for modeling the descriptive perspectives.

Dimension X_2 (nearness) is dedicated to predicated descriptions, and based on that, all its elements belong to the $[-1, +1]$ space located between the two extreme describers -1 and $+1$. This dimension is foundational, as the atomic learnability classifies concepts based on the X_2 predicted comparability. The paper dedicates a group, named $G_{description}$ or in short Γ, for this dimension. For any element $a \in [-1, +1]$, the inverse of that element is $a^{-1} = -a$. Nonetheless, to guarantee the closer property of the group's binary operator, the addition operator is defined as: $\forall a, b \in \Gamma$, $a+b = max(a, b)+{}^{|a-b|}/_{max(a,b)}$. Every notion in the paper comes in dual, and in the learnability section the dual of the $max(a, b)$ switches to $min(a, b)$, and the binary operator may be reinstated as: $\forall a, b \in \Gamma$, $a+_{<}b = min(a, b)-{}^{|a-b|}/_{min(a,b)}$. The two operators $\wedge_{<}$ and $\wedge_{>}$ are dedicated to these two variants and are described in more detail in the chamber geometry section.

The Γ group is utilized further in dimensions X_5 and X_6. The balls used to approximate the X_5 dimension may be modeled as $SO(3) \times \Gamma$. X_6 may be modeled as $SO(3)\times\Gamma\times\Gamma$, as the $SO(3)\times\Gamma$ models the balls containing regions of vector fields, and the last Γ defines how attracted-rebelled the vectors contained in each ball, resulting in the final representation $SO(3) \times \Gamma \times \Gamma$ of X_6. Both X_5 and X_6 are best modeled using Fast Fourier Transform (FFT) for analyzing the geometries perceived by the vision system or abstracted by a cognitive system. The changeability of the seven dimensions represents the deductive system, and the seven dimensions, along with their changeability, may be formalized over the following definition:

Definition 1 (the adposition dimensions). *the seven adposition dimensions may be modeled, along with their changeability, using the following symmetric structures:*

- X_0 *Surroundedness: the abstract model is* $SO(3)_{Pos}$*, and the changeability of this dimension is* $\Delta P = (\partial p_0, \partial p_1)$*, the two dimensions needed for navigating the surface of a ball, two DOF.*
- X_1 *Includeness (containment): this dimension is the most sophisticated as it materializes reasoning and the hierarchical* $if - then$ *statement. It is modeled by a cyclic* $C_{n=3}$ *group action over continues spaces by discretizing them into three subspaces based on the three elements* $C_{n=3} = \{1, \iota, \iota^2\}$ *with the relators of* $\iota^{-1} = \iota^2$ *and* $\iota^3 = 1$*. The neutral element* 1 *represents the boundary (the invariant condition of the* $if(1)$ *statement), while* ι *and its inverse* ι^{-1} *represent the insideness and its counter outsideness. The paper dedicates a specially defined group isomorphic to the* $C_{n=3}$ *named* G_{if}*, which, although finite, has elements that are compact continuous subgroups. The neutral element* 1 *is* $SO(3) \times (+1)$*, which is the condition of* if*, then is dually represented by* $\iota = SO(3)^+ \times (-1)$ *and* $\iota^2 = (SO(3)^- \times (-1))$*. The changeability of this dimension is* $\Delta G_{if} = (\partial 1, \partial \iota, \partial \iota^2)$*.*
- X_2 *Nearness: the proposed group for modeling this dimension is* Γ*, and the its changeability is* $\Delta \Gamma = (\partial \gamma_0)$ *, one DOF.*
- X_3 *Directionality: the abstract model is* $SO(3)_{Dir}$*, and the changeability of this dimension is* $\Delta D = (\partial d_0, \partial d_1, \partial d_2)$ *, three DOF. Therefore* $SO(3)_{Dir}$ *semantically is different from* $SO(3)_{Pos}$*.*
- X_4 *Directional relatedness (modeling swarms): the abstract model is* $(SO(3) \times SO(3))$*, and the changeability of this dimension is* $\Delta(D \times D) = ((\partial d_0, \partial d_1, \partial d_2) \times (\partial d_0, \partial d_1, \partial d_2))$*, six degrees of freedom.*
- X_5 *Spatiality Directionalized: the abstract model is* $(SO(3) \times \Gamma)$*, and the changeability of this dimension is* $\Delta(SO(3) \times \Gamma) = ((\partial r_0, \partial r_1, \partial r_2) \times (\partial \gamma_0))$*, , four DOF.*
- X_6 *Directionality spatialized: the abstract model is* $(SO(3) \times \Gamma \times \Gamma)$*, and the changeability of this dimension is* $\Delta(SO(3) \times \Gamma \times \Gamma) = ((\partial r_0, \partial r_1, \partial r_2) \times (\partial \gamma_0) \times (\partial \gamma_0))$*, five DOF.*

Exemplar Abstraction Compositions for Modeling Adpositions' Phenomena: Composing the adposition abstract typologies may grow in complexity to model any phenomenon. Figure (3, A and B) represents the abstract typology of the adposition ON along with different instantiations of the same type to reflect different interpretations. These instantiations are defined in the next section as the chamber geometry.

The "ON" phenomenon may be modeled as a spatial relatedness between two elements over the positional $SO(3)_{Pos}$ group along with directionality of support, modeled by the $SO(3)_{Dir}$ group, and the value of that supporting load, modeled by Γ. Therefore, saying that a is on b always maintain the typology of $SO(3)_{Pos} \times SO(3)_{Dir} \times \Gamma$ but may be instantiated to reflect specific variants of "On", e.g., a is ON a table b, a is ON a wall b, or a is ON a ceiling b. Figure (3, C) summarizes the possibilities for modeling the different natural languages' adpositions classified according to their semantics.

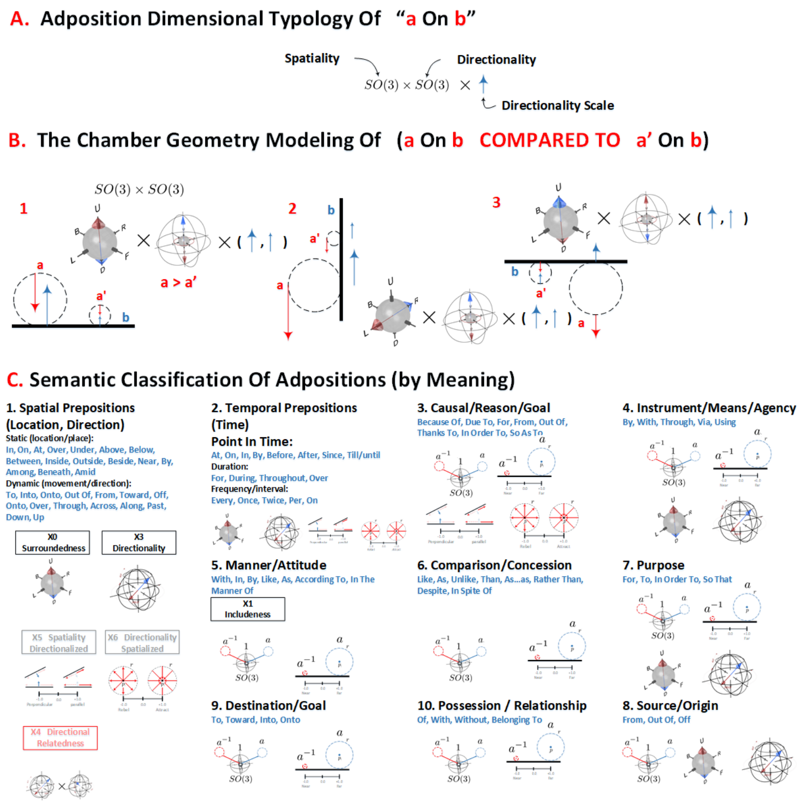

Fig. 3. A) the typology of the ON proposition. B) three different instantiations of the same typology of ON for its meanings by descriptive comparing (chamber geometry) the two entities a and a' over $SO(3)_{pos} \times SO_{dir} \times X_2(scalar - nearness)$. C) exemplar utilizations of the abstract adposition dimensions for assigning semantics to the different natural language's adpositions.

3.2 Defining the Chamber descriptive geometry

Classic geometry studies the measurements of shapes embedded in Euclidean spaces, including distances, angles, area, and volume, as well as their similarity/matching conditions, and their transformative operations, such as scaling, rotation, and stretching. Geometric variants may include Euclidean geometry, which is axiomatized over five axioms, and non-Euclidean geometric variants, which forget any of these axioms. Nonetheless, Modern geometry theorizes these scalar attributes as functions defined over algebraic systems, which implies intrinsically defined geometries, e.g., Manifolds, that are intrinsically well-defined without the aid of extrinsic embedding in Euclidean spaces.

The proposed chamber geometry is an instantiation of the abstract typologies defined by the adposition dimensions (see Figure 2). For example, $a \in SO(3)_{Pos}$ may represent a specific direction a, e.g., north, or $a \in G_{if} \times$ where a may describe a specific statement G_{if} as fast or slow Γ. These instantiations may be furtherly materialized, e.g., as balls $B(p, r)$ or as linear distances, as measurements of physical entities, which may be constructed as 3D scenes to realign the atomic propositions with gamified simulation engines. This minimalistic definition supports physical modeling, e.g., textures, shapes, geometric patterns, complex 3D scenes etc., and rich mental representation. This abstract definition of the chamber geometry represents comparable states, which are the core of the atomic learnability.

The changeability of these states, which may be dually constrained vs. allowed spaces, consumes what the paper introduces as the deductive system. For these states to change, they are coupled with forces, or linguistic verbs. The paper represents forces by $SE(3)$ groups, $SE(3) = \left(\begin{smallmatrix} \mathcal{R} & \tau \\ 0 & 1 \end{smallmatrix}\right)$, where $\mathcal{R}$ is the special rotation group $SO(3)$ and $\tau \in \mathbb{R}_3$ is the translation/position vector. Therefore, the $SE(3)$ group, or any of its discrete or continuous subgroups, may be parametrized by six degrees of freedom (DOF), and that is how the proposed atomic manifold may be parametrized (navigated). In the next section, the operators used to compose complex atomic models are defined along with the deductive system and the hierarchical typology, which is a main factor of the nesting $if - then$ stamens and the self-evolving atomic learnable models.

3.3 The Atomic Connectives, their Convertibility, and the Hierarchicality's Definitions

The material perspective is represented by the cross product of the proposed adposition dimensions (3D material propositions). The behavioral perspective is represented by the $SE(3)$ group, which parametrizes the atomic manifold by its $\mathcal{R}$ rotation and τ translation subgroups (3D behavioral propositions). Finally, the inferential perspective observes/examines the relationship between these two perspectives and the *speed* of their interrelated changeability (3D inferential propositions). These three variations collectively construct the atomic propositions. The inferential perspective is represented by the Γ group, specifically the X_2 dimension, which is not surprising, as it is the perspective responsible for conducting descriptive processing, the primary role of any agency.

These three variants come with their own modeling spaces, and each requires a special variant of the connective operators $\bullet \in \{\wedge, \vee, \neg\}$. The material and behavioral perspectives shape reality that is observed and experientially embodied by the inferential perspective. Therefore, the three perspectives may configure the even structure and its actors, $verb_i(argument_0, argument_1, \cdots, argument_n)$. The verb $verb_i$ belongs to the behavioral perspective, while its arguments $argument_j$ belong to the material perspective. Adverbially describing the $verb_i$ or adjectively describing the $argument_j$ turns the event structure into a 3D scene embodied assessment, with the modeling of the atomic describability. Therefore, the connective operators $\bullet$ come in three versions, which are $\bullet$ for the material space, $\bullet^{\circlearrowleft}$ for the behavioral space, and $\bullet^{\mapsto}$ for processing the inferential space. These connective variants are defined as follows:

Definition 2 (the atomic connectives). *all the three perspectives are represented by mathematical group structures* $\{G, +, -\}$ *and the connective* $\{\wedge, \vee, \neg\}$ *operators are defined using the group operators as follows:*

1. *the* $\wedge$ *connective maps to the* $+$ *binary operator of each corresponding group.*
2. *the* $\neg$ *connective maps to the* $-$ *unitary inverse operator of each corresponding group.*
3. *the* $\vee$ *connective needs an argument,* $\vee(condition_i)$*, and it returns the* $\forall(condition_i)$ *quantifier if the condition is met by the two operands, the* $\exists(condition_i)$ *if the condition is met by any of the two operands, and* $\nexists(condition_i)$ *if none of its operands meet the condition.*

It is important to note that the atomic proposition of an embodied event structure requires a buffer of a 3D scene that may be, in parallel to the algebraic-geometric processing, gamified over any gaming engine. The capacity of the event structure buffer may differentiate the specs between different agents. Another important note is that using the group representations for defining the connectives would imply that the atomic connectives $\{\wedge, \vee\}$ are ***non-commutative***, but still ***associative***. Join negation $\neg\vee(condition_i)$ is $\vee(\neg condition_i)$, and meet negation $\neg(a_0 \wedge a_1)$ is $(\neg(a_1) \wedge \neg(a_0))$, because $\wedge$ is not commutative and its parallel group operation $-(a_0 \bullet a_1)$ is $(-a_1 \bullet -a_0)$.

The Equivalence $\Leftrightarrow$ and Implication $\Rightarrow$ Connectives: These two operators are foundational to the first/higher-order logic, and they are the main factor for formulating language semantics using the functional representation of these languages. The atomic implication $\Rightarrow$: $(observer_i, P_j) \rightarrow P_{j'}$ expects an $observer_i$ and atomic propositions P_j as arguments, and translates P_j to the atomic $P_{j'}$ proposition, as assessed by the observer $observer_i$. The equivalence $\Leftrightarrow$: $(observer_i, P_j, P_k) \rightarrow (\mathbf{R}^3 \in Perspective_{l\in\{0,1,2\}})$ expects an extra P_k argument, which is the atomic proposition that is compared to P_j by the $observer_i$. The comparison returns the closeness between the two atomic propositions for each of the three perspectives. These two operators are foundational for communication between different atomic robots, e.g., a fly-sized robot with a factory-arm-scaled robot. They require initial guessing about the bodily properties of the $observer_i$, and they may be accurately transformed from an observer to another because of the group modeling of the seven adposition dimensions.

Defining the Convertibility of the Atomic Connectives: Behavior and state conceptually exist because they can be observed by an experiencer, and observing implies the describability of these two interrelated domains. The description is the observed infinitesimal changeability of an atomic proposition $\Delta P = (\partial p_0, \cdots, \partial p_n)$ measured by the speed of change $\overrightarrow{\Delta P} = P_{t+1} - P_t$.

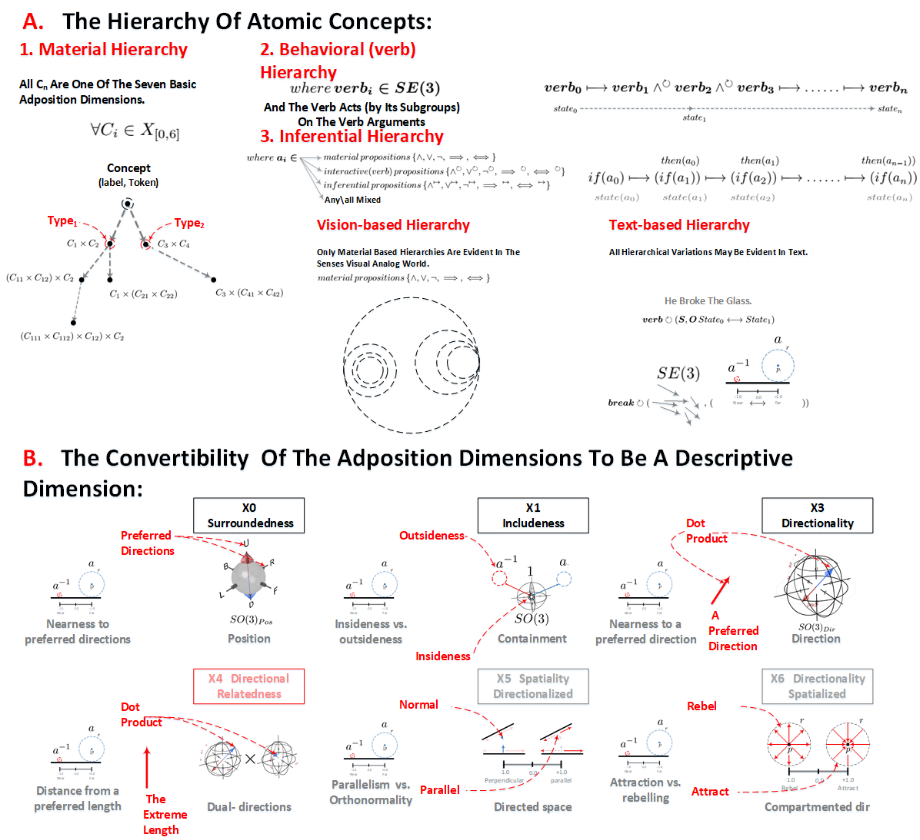

Fig. 4. A) the three versions of hierarchicality. B) the convertibility between the six perspectives and the descriptive X_2 dimension.

Nonetheless, these infinitesimal vectors are scaled appropriately as group elements of Γ, which is the group used by the paper to represent the inferential perspective. Therefore, the types of behavior (verb) and states (verb arguments) have to be convertible to the Γ group, the way the third inferential perspective does, which may be defined as follows:

Definition 3 (the atomic convertibility). *the group representation G of the seven adposition dimensions may be converted to the Γ group by defining the following $\varphi : G \to \Gamma$ map:*

1. *select an extreme element $g_1 \in G$, and its inverse g_1^{-1};e.g., right and left, front and back, or up and down; and map them to the extreme element of Γ, $\varphi : g_1 \to +1 \in \Gamma$ and $\varphi : g_1^{-1} \to -1 \in \Gamma$.*

2. *select a group element $g \in G$ that lies between the two extremes, e.g., for right and left select front or back, and map it to the identity element e, $\varphi : g \rightarrow e \in \Gamma$ map.*
3. *measure the distance, Euclidean or geodesic, for any group element g_i and the extremes, $d = min(dist(g_i, g_1), dist(g_i^{-1}, g_1^{-1}))$, and normalize it $norm(d)$ on $norm(d) \in \Gamma$ in the direction of the least extreme $\varphi : g_i \rightarrow [-1, +1] \in \Gamma$*

In reality, a right-handed person may have a different convertibility system than a left-handed person, and so a robotic agent is expected to have a unique convertibility system matching its body's configurations. Each of the three variants of the atomic connectives $\{\bullet, \bullet^{\circlearrowright}, \bullet^{\mapsto}\}$, see Definition (2), are further dually defined, e.g., $\{\bullet_{<}, \bullet_{>}\}$, to match the binary operation of the Γ group, see Definition (1) and Figure (4, B).

Defining Hierarchicality: Hierarchy is the foundation of the generalization and abstraction processes, two main traits of human intelligence. Similarly, for a recursive atomic concept to be meaningfully explainable, it has to exist in fairly simple contexts, and then, it ascends in complexity by explicating more details one step after another.

Similar to the atomic connectives, there are three interpretations of hierachicality, which are the material hierarchy id defined syntactically over the atomic propositions (part-to-whole), the behavioral hierarchy is defined over a series of behaviors affecting a single type of states where the effects propagate over the path of actions, and finally, the inferential perspective's hierarchy as a montonically boosted, or inversely decayed, infinitesimal changeability over a single behavioral or material type, see Figure (4, A).

In all three variants, an effect of a starting point (the root) consistently continues to propagate through a series of its successors, and such a hierarchical path ceases to exist once that propagated effect diminishes. The three interpretations of the path of hierarchical series are defined as follows:

Definition 4 (the atomic hierarchicality). *Hierarchy is an effect $\mathcal{E}$ that is initiated and influenced by a source, the root of the hierarchy $h_0 \in \mathcal{H}$, and consistently propagates to its successors. The hierarchical path $\mathcal{H}$ ceases to exist once the effect is no longer observable. The three perspectives' interpretation of hierarchicality is as follows:*

1. *the material hierarchy: for any given atomic proposition, $P^0 = (p_0 \bullet p_1 \bullet \cdots, \bullet p_n)$, where $\bullet \in \{\wedge, \vee, \neg, \Leftrightarrow, \Rightarrow\}$, and all their variants, see Figure (4, A.1), hierarchy $\mathcal{H}$ is a series of recursive substitutive process by defining a map φ that adds more details by substituting any symbol p_i by an atomic proposition, $\varphi : p_i \rightarrow p_i \times (p_{i0} \bullet p_{i1} \bullet \cdots, \bullet p_{im})$, which yields a new more detailed atomic proposition $P^1 = (p_0 \bullet \cdots \bullet (p_i \rightarrow p_i \times (p_{i0} \bullet p_{i1} \bullet \cdots, \bullet p_{im}) \cdots \bullet p_n)$, where the new added parameters of p_{ij} are dependent on the p_i parameters. The substitutive process may go on, in such a detailing manner, from proposition P^0 to P^k.*

2. *the behavioral hierarchy: given a state s, or a state derived from it using material hierarch, the hierarchy $\mathcal{H}$ is a series of n verbs, $verb_0(s) \mapsto verb_1(s) \mapsto \cdots \mapsto verb_n(s)$, where each verb $verb_i$ maintaining the effects of its prior nodes $verb_0$–$verb_{i-1}$, see Figure (4, A.2).*
3. *the inferential hierarchy: either fixing a state s and constructing a behavior hierarchy, or fixing a verb $verb_i$ while changing its arguments,collectively named d, a single describer type $\triangle$ constructs the hierarchy $\mathcal{H}$ as a series of n describers, $\triangle_0\,(d) \mapsto \triangle_1\,(d) \mapsto \cdots \mapsto \triangle_n\,(d)$, where each $\triangle_{i+1}\,(d) > \triangle_i\,(d)$, see Figure (4, A.3).*

The material hierarchicality may be used to define the behavioral hierarchy, while the inferential hierarchy may be defined over any of the other variants of hierarchicality. These simple hierarchical variants, which build on certain fixed arguments, are used as building blocks of the atomic learnability section, which hierarchically explains a relationship between any two concepts. The semantics of these hierarchies is grounded using the $if-then$ proposition, as detailed next.

3.4 The Hierarchically Nesting ($if - then$) Statement, the Deductive System, and the Type System

A verb relates its state-based arguments (the arguments and adjuncts), and it parametrizes the event structure based on its subgroups' translational and rotational parameters [14; 15], see Figure (5). The $if - then$ statement spans beyond the boundary of a single event structure, and in doing so, the $if - then$ statement interlinks the different event structures' constituents. Therefore, the $if-then$ statement creates the meaning of concepts by constructing relativistic networks of contextualized concepts, see Lemma (reflemma:relativistic).

The $if(condition_i)$ and its dual detailing conclusions, $then^+(conc_i)$ and $then^-(conc_i)$, is a practice of the atomic hierarchy. Using Definition (4), it is $if(P^0)then(P^1)$. Meaning that knowing the condition $condition_i$, then the $conclusion_i$ is a possible detailing, or else, knowing the conclusion $conclusion_i$, then the condition $condition_i$ is a possible abstraction. These hierarchically related possibilities should behave similarly on a much deeper hierarchical path of $if - then$ detailing propositions. The use of the nesting $if(condition_i) \mapsto \{then^+(conclusion_i), then^-(conclusion_i)\}$ is used extensively in the next learnability section for modeling and querying local conceptual simulations of event structures.

The Deductive System: The proposed deductive system is the three interpretations of hierarchicality. It represents the three interpretative variants of the proof systems that may be grounded by physics and mathematics. For physics, fixing parameters of an experimentation to collect substantiated results may be compared with the fixed states for behavioral hierarchies, or the fixing of either behaviors or states for defining instances of inferential hierarchies. Similar

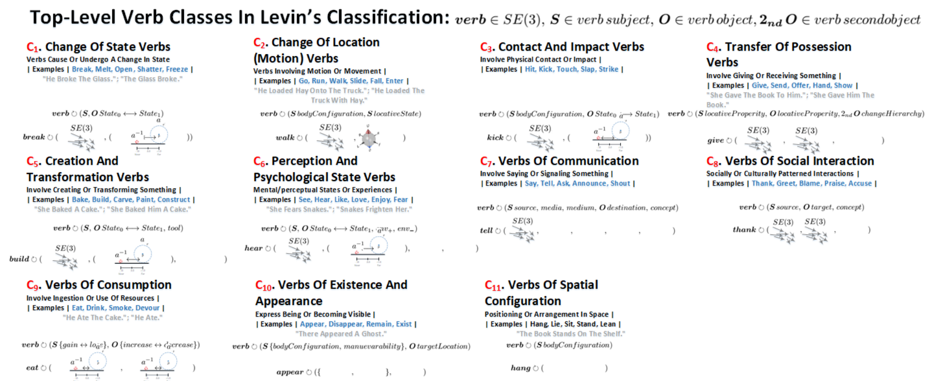

Fig. 5. The eleven classes of verb structures populated by atomic propositions as verb arguments.

arguments may be applied to the mathematical axiomatization system of mathematical structures, which may yield interesting results concerning how human agents understand mathematics.

The proposed atomicity defines relations between different entities using the atomic type system, the seven adposition dimensions, which are symmetric. Nonetheless, any references that may be deemed asymmetric would still leverage the symmetry of the atomic model. Part of the assumptions of the proposed atomicity is that the seven adposition dimensions and their cross products may cover all the topics needed to model any material or immaterial entity that may be otherwise thought to be asymmetric.

The Type System: The atomic propositions model event structures, formalized semantic modeling of natural language. The proposed 3D atomic connectives $\cdot \in \{\wedge, \vee, \neg\}$are meant to model the material states as arguments $Args_{n\in N}$ of verbs, e.g., $verb^{\circlearrowright}(Args_{n\in N})$ [14; 15], see Figure (5). The behavioral and inferential atomic propositions use the connective variants $\cdot^{\circlearrowright}$ and $\cdot^{\mapsto}$, which may be semantically represented by $describe^{\mapsto}(verb^{\circlearrowright}(s, o_0, o_1, args_{n\in N}))$. Based on that, the types and their respective atomic connectives are as follows.

1. the 3D material atomic propositions: they are represented by the seven adposition dimensions and their direct cross products to model states that are used as arguments for verbs. They use the $\cdot$ connectives.
2. the behavioral atomic propositions (verbs): although any $verb^{\circlearrowright}(Args_{n\in N})$ is abstractly represented by an $SE(3)$ group to parametrize its arguments, the $verb^{\circlearrowright}$ type is defined by its arguments, as the role of the verb is to link and parametrize different materialistic states. The $\wedge^{\circlearrowright}$ connective expects two verb operands of the same type.

3. the inferential atomic propositions (descriptions): the $describe^{\mapsto}$ type compares the speed of change of the parametrized event structure, based on subjective observation or experimentation, and normalizes this comparison over a $[-1, +1]$ scale. Or in other words, $describe^{\mapsto}(verb^{\circlearrowleft}(s_t, o_t, \cdots)) \longmapsto verb^{\circlearrowleft}(s_{t+1}, o_{t+1}, \cdots)$. The $\wedge^{\mapsto}$ connective expects two operands of the same type.
4. the three versions of hierarchicality: they are composed under the same roles of their respective atomic connectors.
5. the nesting $if-then$ statements: they are responsible for the invariant binary classification of the learnability space.
6. finally, the query types $q \in \mathcal{Q}$: they are the only types, additional to the basic adposition deimnsions of Figure (2), that may define direct cross products $q = (C_0 \times C_1)$ of two concepts in response to queries. In the next learnability section, approaches for automatic type definitions for latent conceptual spaces are proposed.

3.5 The Atomic Manifold's Modeling of State, Change, and Describability

Equation (1) represents the set of concepts, which may be compared to LLM's tokens, related to each other. Over this matrix relatedness, atomic propositions may be structured between any two concepts. Figure (6) depicts a visualization of how such atomic propositions (states) may be dynamically changed by parametrizing verbs, which themselves are different manifestations of the set of concepts. The possibility that the changeability of different hierarchical states may further substantiate different typologies signifies that the atomic manifold may be so big that it can't exist as a global phenomenon, as the whole known and yet to know sentence variants may be modeled over the state-change of the atomic manifold.

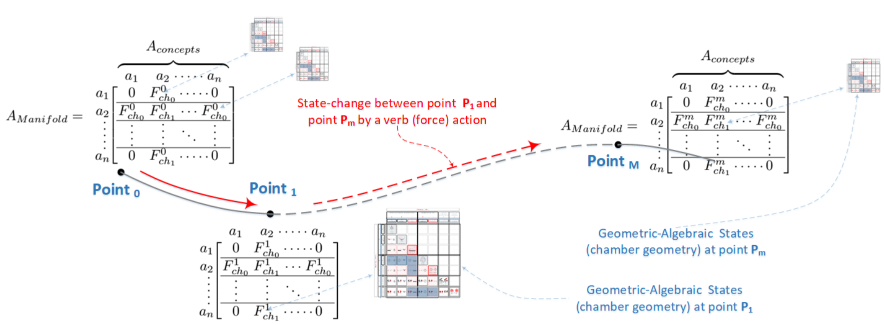

Fig. 6. an illustration of the components of the atomic manifold and its verb's paramerization to smoothly navigate across the memorized states

$$
A_{Manifold} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \overbrace{\begin{array}{ccc} a_1 & a_2 \cdots\cdots & a_n \end{array}}^{A_{concepts}} \left[\begin{array}{cccc} 0 & F^0_{ch_0} & \cdots\cdots & 0 \\ \hline F^0_{ch_0} & F^0_{ch_1} & \cdots & F^0_{ch_0} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & F^0_{ch_1} & \cdots\cdots & 0 \end{array}\right] \quad (1)
$$

Therefore, the atomic manifold only exists as a local phenomenon, and these local phenomena are recorded over the hierarchical atomic graph, which certifies the atomic graph as a differential graph memorizing snapshots of the local reasoning. Therefore, generalization or detailed specification are equally treated as a local phenomenon, which implies the recursive reality of the atomic concept. In fact, the whole manifold is a concept, or equivalently, a linguistic word, much like a self. Learnability of the next section shapes the locality of the atomic model, and a better definition of the atomic manifold is given in the coming "algebraizing the atomic manifold" subsection.

4 Atomic Learnability

Classification , ***regression*** , and ***clustering*** are the main tasks machine learning (ML) algorithms perform. ML builds on two consecutive processes. The first is *learning* the weights of a model, while the second is applying the learnt model in *inference* applications. Nonetheless, the two processes are unrelated as the inference process is unaware of the learning process, and there is no mutual developmental interaction between them.

Contrary to that, the atomic concept conducts ***local*** algebraic-geometric simulated comparability between two atomic statements, and as a result of that comparison, the proper explainable classification is defined, the long-term atomic graph memory is shaped, and the trajectory for a better, closer value may be guessed for infinitesimal enhancements. This classification is regressively defined, and classification and regression are entwined. To compare two atomic concepts, 1) they are assumed to be of the *same type* (typological requirements), and then, 2) to be *descriptively scaled*, as a predicate, for more (greater) against less (smaller) comparability.

The atomic modeling builds on representing materiality/conceptuality, using the seven adposition dimensions types and the chamber geometric materialization (3D atomic propositions), as contributors of an **event structure**. Consequently, similarity is analogically assessed between two atomic concepts based on their *semantic roles* in event structures. Nevertheless, event structures imply the ongoing dynamics of change and interactions, and consequently, tracking how concepts are related over the course of behavioral processes constructs explainable relationships between any concepts involved in that course of actions. Based on that, the following two mechanisms are facets of the atomic learnability:

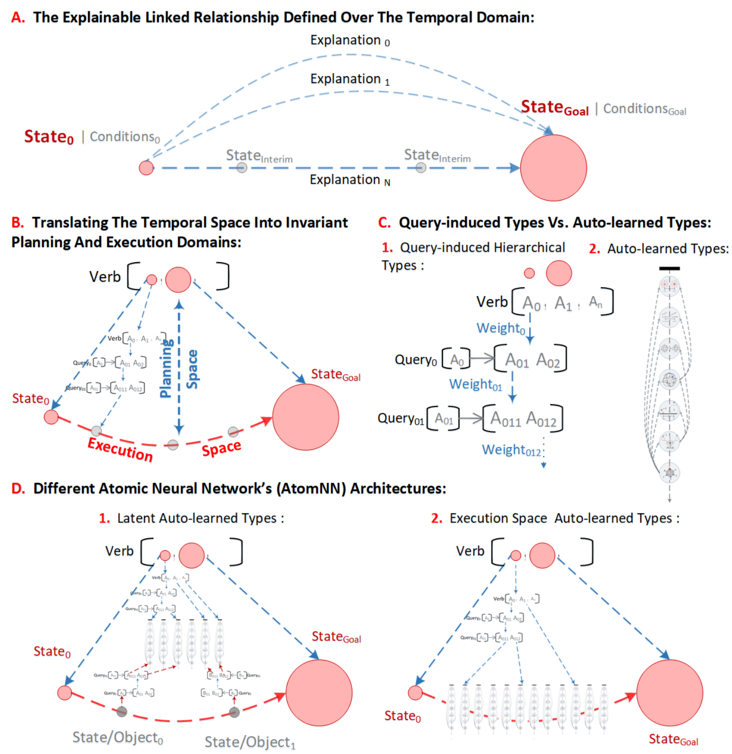

Fig. 7. A) atomic learnability defined as the best explainable paths over the temporal domain that link the starting state ($State_0$) to the goal state ($State_{Goal}$). B) the atomic neural network (AtomNN) defined over the invariant planning and execution domains. C) the two methods for defining the network of queries of the AtomNN. D) the AtomNN architectures building on the predefined set of queries or auto-defined queries.

1. similarity conditions: given two concepts, what are the conditions needed to make them similar? Or inversely, what are the consequences of considering any two concepts similar?
2. an explainable path of actions for linking two states: given two concepts, what are the possible paths of actions that explain a relation between these two concepts? Or inversely, given a path of actions that explains a link between two concepts, what are its implications?

Therefore, stating that ($concept_i \equiv concept_j$) , given the condition that $explanation_k$, or the two concepts are related ($concept_i$) $\rightarrow explanation_k \rightarrow$ ($concept_j$), because of ($\rightarrow explanation_k \rightarrow$), are a conclusion of an atomic reasoning (searching) process, see Figure(7, A).

Although these two tracks may sound different, both of them are merely an ***explanation*** of a binary relation between two states, in the first it is equivalence, and in the second it is context-based relatedness. Meaning that atomic learnability explains a relation between a starting state $State_0$ and a goal state $State_{goal}$, and with constraining conditions, $(State_0 \,|\, Condition_0) \rightarrow explanation_k \rightarrow (State_{goal} \,|\, Condition_{goal})$.

For example, a hammer may be related to a chair over the paths of: a hammer is used to nail a chair, a hammer may be put on a chair, or the hammer's handle is made from the same chair's material, etc. although all of these explanations are valid, some may give better explainability than the other in a specific context. Similarly, a car and a boy may be deemed similar under the description of speed, as both coexist in the same building, etc. The rule is that a relationship linking two concepts is explainable.

This simple notion of explaining a link between two states may prove to an efficient reasoning approach that unifies a multitude of cognitive and thinking processes, e.g., planning, storytelling, criticizing, discovering, searching, mimicry learning, puzzle solving, one-shot learning, spatial navigation, or designing.

In the following subsections, a simple atomic proposition, e.g., a $verb_i$ statement, that relates the two states is proposed as the root for a consequent detailing process for constructing explainable paths relating the two states. Following that, an Atomic Neural Network (AtomNN) is proposed, and an evolutionary optimization process is proposed to hierarchically train the differential parameters of the AtomNN's hierarchical explanation. Constructing and querying the AtomNN is conducted invariably over a nesting $if - then$ statements.

4.1 Atomic Learnability as Explainable Paths for Linking Two States

Event structure-based similarity is subject to general or specific features. The general features are the three perspectives: the material, behavioral, and inferential (observable) perspectives, along with the Beforeness (the prerequisites) and afterness (the implications or consequences) of the event structure. On the other hand, the specific features are the semantic roles played by the contributors of any event, e.g., subject, object, source, purpose, manner, etc. Noting that two of the general features, beforeness and afterness, are evident in the event arguments, e.g., $Object_t$ and $Object_{t+1}$, which may weaken the need for explicit usage of Beforeness and afterness of events. Nonetheless, the paper may use these two features for clarity and simplicity, and to highlight the notions of prerequisites and implications (see Figure (8) B and C).

A compact representation of the atomic sentences is:

$$describe^{\mapsto}\,(verb^{\circlearrowright}\,(\{S^{\mapsto}, S\}, \{O_0^{\mapsto}, O_0\}, \{O_1^{\mapsto}, O_1\}, arg_{n \in N}))$$

Where $\{S\,,\,O\}$ are inanimate subject and object, respectively, while $\{S^{\mapsto}\,,\,O^{\mapsto}\}$ are the animate versions. Assigning any of the three variants $describe^{\mapsto}$, $verb^{\circlearrowright}$, and the verb arguments $arg_{n \in N}$ to the $State$ of the explainable course of actions formalizes the explainable paths as follows:

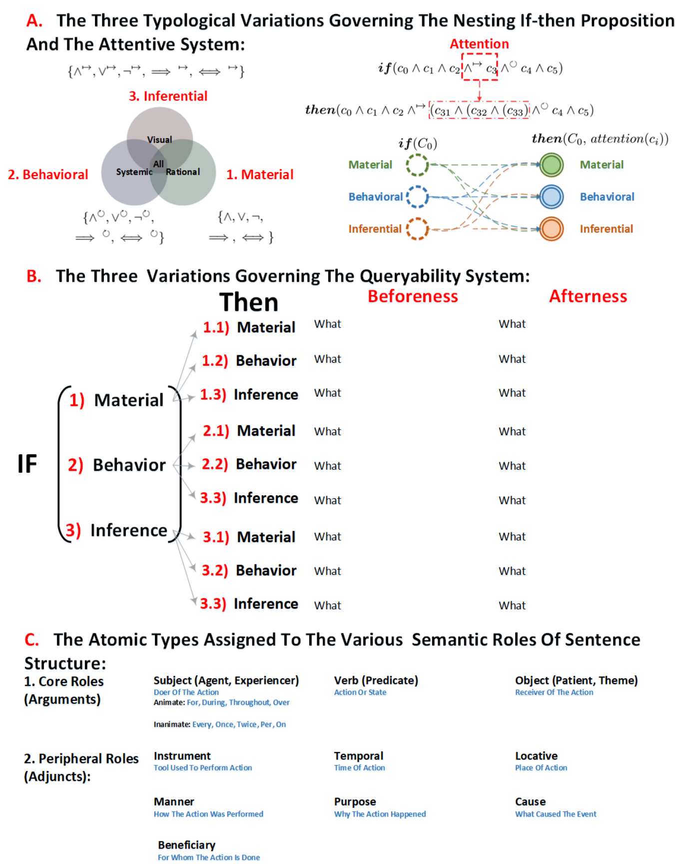

Fig. 8. A) the nesting $if-then$ proposition and the attentive queryabilities. B) detailed queryabilities. C) the various atomic types of the contributors to the even structure.

Definition 5 (the explainable path for linking two states). *for* $State_0$ *,* $State_{goal}$ $\in \mathcal{S} = \{describe^{\mapsto}, verb^{\circlearrowright}, arg_{n\in N}\}$*, an explainable relation* $State_0 \longmapsto State_{goal}$ *is a series of steps proceeding from the start state* $State_0$ *to a goal state* $State_{goal}$*.*

4.2 Defining the Trainable Atomic Models

Learning an explainable path for $State_0 \longmapsto State_{goal}$ starts from the queryability of how $State_0$ and $State_{goal}$ may be interpreted. This reformulates the abstraction of explainable paths, see Definition (6), as a pair of $s \in \mathcal{S} and q \in \mathcal{Q}$, for states s with the coupled set of queries q. For each query $q \in \mathcal{Q}$, there are expected to be different paths of explanation, as defined in the type system for how queries may construct the dimensions of the types.

Although coupling states with queries enriches the explainable path and makes them communicative and meaningful, the iterative practice of queryability, e.g., querying the earlier queries, may end up with a complex space that is harder to construct. Consequently, the paper proposes deducible explainable paths as simple building blocks for the more complex paths. The ***deducible explainable paths*** reutilizes native atomic types as simple explainable paths, and that may include any of the following two types:

1. both $State_0 \in \mathcal{S}$ and $State_{goal} \in \mathcal{S}$ are collaborators in a single event. Meaning that the explainable path $State_0 \longmapsto State_{goal}$ is merely an event, a single verb.
2. both $State_0 \in \mathcal{S}$ belong to one of the three variants of hierarchicality. Meaning that $State_0$ is a simplification of the more detailed $State_{goal}$ of the same hierarchy, the explainable path $State_0 \longmapsto State_{goal}$ is merely a hierarchy.

Any algebraic-geometric construction of explainable paths will depend on these two simple, deducible explanations. Additionally, as a complement to the *deducible explainable paths*, event-induced (context-based) concepts that are deemed similar to $State_0$ or $State_{goal}$ may be used for knowledge transfer their explainable paths.

Nevertheless, for an explained link between two states to evolve into a complex, detailed path, it needs a simple statement that summarizes how the two states may be related. Based on that, the first theorem of the paper assures the existence of a single statement linking the two states and shapes the expected detailed explanation.

Theorem 1 (the atomic learnability). *given a starting state $State_0$ and a goal state $State_{goal}$, any explainable relationship $\mathcal{L} = State_0 \longmapsto State_{goal}$ can be summarized using a deducible explanation as the root of the explainable path, and the hierarchical detailing of the deducible proposition produces the finer more detailed explanation of the relationship $\mathcal{L}$.*

Therefore, a simple deducible path, e.g., a verb or a hierarchy linking $State_0$ and $State_{goal}$, is the first step for learning the detailed explanations derived from this simple atomic proposition. Consequently, the temporal explainable paths linking the two states, as illustrated in Figure (7), are produced by hierarchical detailing of the summarizing atomic proposition.

The drawback of the given definition (6)is that the explanation path $\gamma(\tau \in [0,1])$ is situated over a temporal domain $\lambda_{p\in P}$, where $\gamma(\tau \in [0,1]) \mapsto \lambda_{p\in P}$, and $\gamma(0) \mapsto State_0 \in P$ and $\gamma(1) \mapsto State_{goal} \in P$, which implies a concrete temporal instance that is not ***invariant*** and not ***generalizable***, and hence not *communicate-able* between different scaled robots and natural agents. Consequently, the given definition (6)of atomic explainability needs to be translated into invariant atomic domains.

Figure (7, B)represents the paper's proposal for translating the concrete temporal domain of any explainable path into two invariant domains, the first is called "execution domain", which represents the temporal explainable path by abstract atomic propositions. The second invariant domain is called "planning domain" and it represents the paths from the summarizing verb to the execution space. The paper uses the hierarchically nested $if - then$ propositions to model these paths.

In summary, Definition (6)defines a temporal series of actions to explain a relation between two concepts. Theorem (1) proposes a simple summarizing proposition, a verb, that links the two entities, and accordingly, shapes the explainable relationship. Finally, two invariant *execution* and *planning* spaces are proposed to invariantly represent an explainable model. Based on that, the following is a definition of the atomic neural network (AtomNN) that may equally hold the symbolic and connectionist traits:

Definition 6 (the Atomic Neural Network (AtomNN)). *the AtomNN explains a relationship between two states* $State_0$ *and* $State_{goal}$ *based on a simple deducible path* $State_0\,,\, State_{goal}\ \in \mathcal{S} = \{\, describe^{\mapsto},\, verb^{\circlearrowleft},\, arg_{n\in N}\}$. *The explanation is defined over the following two invariant constituents of the AtomNN:*

1. *the planning domain: this domain resembles a discourse or a thinking process that emits from* $s \in \mathcal{S}$ *and emerges towards the execution course of action using series of queries. The queries are represented by nesting* $if - then$ *propositions.*
2. *the execution course of action: it prioritizes entities, concepts intuited from* $s \in \mathcal{S}$ *or objects found in the surrounding environment, over a course of action based on the processing of the planning domain.*

In the next section, training the AtomNN using evolutionary optimization approaches is proposed. The optimization approaches are applied to group-inherent equalities using the nesting $if - then$ propositions.

4.3 Defining the Architecture and Optimization Techniques of the Atomic Neural Network (AtomNN)

Optimization is a search over a differential space for elements that guarantee optimality/minimality under given differential conditions. This differentiable space is either embedded in an ambient Euclidean space or, rather, a manifold that is intrinsically defined. For ML models, e.g., DNNs, the differential model and the cost functions are predefined. For the AtomNN, the model is dynamically defined as an atomic type (diff. topological type). The AtomNN's type is dynamically structured as the nesting queries that probe different facets of the summarizing verb $verb^{\circlearrowleft}_{sum_i}$ of the linked states, $verb^{\circlearrowleft}_{sum_i}(State_0, State_{goal}, arg_{n\in N})$. The hierarchically structured queries are represented with their infinitesimal changeable parameters, $\varDelta_i = \sum_{n\in N} \partial_n$, see Definition (1) for the differential parameters of the seven adposition dimensions. The ***execution*** and ***planning*** domains and their relatedness with the $verb^{\circlearrowleft}_{sum_i}$ ***summarizing proposition*** are deliberated in the following subsections.

The Planning Domain: The query's hierarchicality is defined over weights ($weights_i$) inherited from a parent query to its following nesting queries, see Figure (7, C.1). Consequently, the planning domain of the AtomNN is defined as a set of hierarchical differentials that are linked by the proper weights. The hierarchical parametrization ($weights_i$) is the trainables of the planning domain, and they model the extents to which the event structure's parametrizations $verb^{\circlearrowleft}_{sum_i}$ correlate with the changeability of the execution domain, which is presented next.

The Execution Domain: The execution domain may be visualized (imagined) as navigation paths that the surrounding environment's objects Obj_{env_i} may follow. The navigation path defines the series of actions $verb^{\circlearrowleft}_{exec_j}(Obj_{env_i})$, where $verb^{\circlearrowleft}_{exec_j} \in SE(3)$ affects any/all of the surrounding environment's objects. This physical manipulation of material objects may simulate, as well, the immaterial cognitive processes. That is because the environment's objects are represented by atomic propositions, and according to the paper's axiomatization, may equally model the material and the immaterial entities.

For any object Obj_{env_i}, there is a map, $\varphi_i : State_0 \mapsto Obj_{env_i}(State_0)$ and $\varphi_i : State_{goal} \mapsto Obj_{env_i}(State_{goal})$, between the two states of the explainable path and corresponding states of the physical environment's objects. Therefore, the φ_i interprets the Obj_{env_i} states that may be changeable over the course of action. Meaning that, each object Obj_{env_i} in the execution domain has, at least, to be paramerized by an intransitive verb $verb^{\circlearrowleft}_{exec_j}(Obj_{env_i})$, and this verb $verb^{\circlearrowleft}_{exec_j}$ has to exist even if its label may be unknown, e.g., a latent unrecognized/novel verb to the agent's knowledgebase. For relating difffernt objects of the execution domain, transitive verbs are used along with the environment's arguments, e.g., $verb^{\circlearrowleft}_{exec_j}(Obj^{\mapsto}_{env_i}, Obj^{\mapsto}_{env_j}, Obj_{env_k}, Args_{n\in N})$, where $Obj^{\mapsto}_{env_i}$ is an animate object, while Obj_{env_k} is an inanimate.

Interrelating the Planning and Execution Domains: The goal of AtomNN is to relate the observed *verb-state* speed of changeability of the *summarization proposition*, $verb^{\circlearrowleft}_{sum_i}$, $(State_0, State_{goal}, arg_{n\in N})$, with the *execution domain states* defined by the map φ_i for each object Obj_{env_i} of the execution domain. The hierarchical queries' parameters $\Delta_{i\in I}$ link the *summarization proposition* and the *execution domain states*. The role of the hierarchical queries is to correctly orient the conversation/thinking processes with the execution course of action, in response to the parametrization of $verb^{\circlearrowleft}_{sum_i}$. Consequently, the interrelatedness between the hierarchical queryability and the execution course of action defines the AtomNN architectures, as detailed in the next section.

AtomNN Architectures: The two factors shaping the architectures of AtomNN are the queryability techniques and the execution path's interrelatedness to the hierarchical queryability parameters, and both of these factors are detailed as follows:

The Queryability Techniques: Queryability defines the typological model of the AtomNN. Queries may be defined either manually, see Figure (8), or automatically, by learning (training) to ask the proper question, see Figure (7, C.2 and D). It is important to note that queryability is a main factor of commonsensical communication, and training a developmental agent to ask the proper questions means properly querying/defining the AtomNN, and that is foundational.

The Execution Path's Atomic Propositions and Their Relationship with the Hierarchical Queryability Layer: The proposed AtomNN is developmental. Meaning that agents may learn to ask proper questions $q \in \mathcal{Q}$ as much as learning to relate, by clustering, different environmental surrounding objects over $verb^{\circlearrowleft}_{exec_j} \in SE(3)$ atomic propositions. Consequently, combinatorial calculations, e.g., $\binom{k}{n}$ for relating k environmental objects out of n environmental objects, may be considered in early developmental phases, or else, proper clustering algorithms should supplement the atomic agents.

For the AtomNN architecture: the execution layer is either agnostic about the specific parameters of the queryability layer (feedforward networking), or else, it is in a one-to-one relationship with all the parameters of Δ_i and $(weights_i)$ of the queryability layer, see Figure (7, E)

Theorizing the AtomNN's Optimization-ability: The numerical optimization of AtomNN builds on two factors: the *group-inherent* qualities and the descriptive *comparability*. Both their quantitative and qualitative aspects are deliberated upon in terms of usability by evolutionary optimization algorithms to train the AtomNN. The section begins by introducing the group-inherent dualistic calculations, along with some AtomNN-driven dualities. Then, different approaches for assessing similarity based on comparability possibilities are discussed. After that, an evolutionary genetic programming algorithm is suggested to benefit from the rich set of duals and comparabilities. The section closes with the theorem of the AtomNN's optimization-ability.

The AtomNN Group-Inherent Optimization Methods : AtomNN is symmetrically represented using group structures of the $if - then$ nesting queries or the atomic propositions. Therefore, for any dual elements a^{+1}, a^{-1}, their multiplication should yield the identity element $a^{+1} \cdot a^{-1} = e$. This fundamental role of symmetry should be numerically maintained by the proposed backpropagation process. The multiplication of the partial differentials of both of the duals is optimized to be as close as possible to the identity element $e = 1$. Although it may be taken for granted that inverses multiply to identities, evolutionary algorithms may mix different manifestations of what may be considered inverses for the generation-based optimization enhancements.

AtomNN-Extractable Duals Although atomic propositions and hierarchical $if - then$ paths are symmetric, with plenty of group-based dualistic processing, this section is interested in the duals related to the AtomNN architecture. The planning domain maintains the top-down versus the bottom-up dualities, while the execution course of actions maintains the duality of the directionality between $State_0 \mapsto State_{goal}$ path versus its inverse direction $State_{goal} \mapsto State_0$. Both of these dualities may be considered independent, and their typological and scalar instances are optimized for the search for a better solution.

Comparability Approaches and Theorizing the AtomNN's Optimize-ability To complete the list of duals, the comparable duality is explicated. Although all the mentioned dualities may be selectively used, this duality is mandatory for the atomic optimization process. To conduct comparability, there has to be a selected extreme $\in \{-1\,,\,+1\}$, and based on that selection, comparability is possible. One positive side of the proposed atomicity is that a verb or a state may be equally turned into a comparable entity, see the atomic convertibility. Comparability means that it is possible to ask a question like “what is it like?” for a behavior or a material to compare to. There are two options to select from to conduct comparability, the first is related to the states and behaviors of the execution domain, from the surrounding environment, while the second is deriving the comparability from the summarizing proposition. Merging these two approaches is the job of the evolutionary optimizer.

Based on that, the atomic optimization-ability may be theorized as:

Theorem 2 (the atomic optimization-ability). *the two planning and execution layers of the AtomNN may co-train under any of the selected dualities* $d_{i\in I} \in \mathcal{D}$*, and using either any/both of the execution-based or summarizing proposition-based comparability. Employing a genetic programming evolutionary optimization algorithm* $\mathcal{G}$*, the atomic optimize-ability algorithm may be defined as:*

1. *define the space of dualities that may be possibly, or impossibly, used.*
2. *define the comparability approach.*
3. *assign these parameters, along with the summarizing proposition and all the nesting* $if - then$ *propositions that either allow or disallow the queryability space.*
4. *assign an acceptable error value for the computed group-inverse multiplications that the evolutionary algorithm may finish processing after reaching.*

4.4 Memory Management

The event structures are open subsets of the differential atomic manifold $\mathcal{M}$ at the continuous neighborhood of the point $p \in \mathcal{M}$, where:

$$p \in verb^{\circlearrowleft}\,(\{S^{\mapsto}, S\}, \{O^{\mapsto}, O\}, arg_{n\in N})$$

Which is a model of the paramerized verb represented at p. These open subsets represent all the verbs in natural language as smooth submanifolds that may algebraically-geometrically be simulated using atomic propositions. These submanifolds are stored as isolated islands in the *atomic graph*. Additionally, the

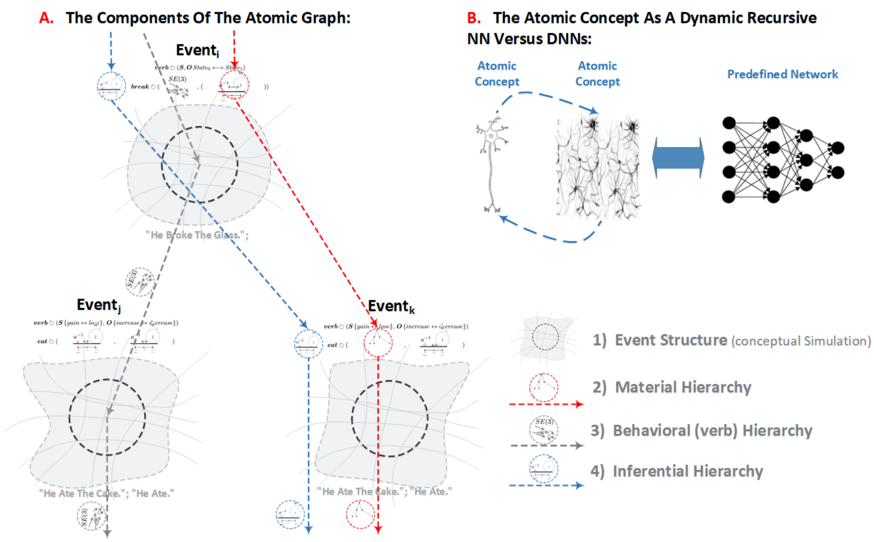

Fig. 9. the elelemnts of the atomic graph, the nodes of the 3D algebraic-geometric atomic simulation and the stored three versions of hierarchicality.

three variations of hierarchicality spanning the arguments of the verbs and the series of explainable paths are stored, as well, in the atomic graph, see Figure (9, A)

The amount of the stored verbs and the deductive hierarchies may vary from one robot to another, depending on the computational capacity of each robot. The long-term memory (the atomic graph) is supplemented with a memory buffer that holds the retrieved chamber geometric simulations of the verbs and the related hierarchies. Additionally, the memory buffer holds the atomic propositions as controlling examples that may command a swarm of atomic agents. The same relatedness between the short/medium-term memory buffer may supplement the multi-system interactions, e.g., vision, motor system, and higher cognitive processes. All these sensorial manifolds may be stored as submanifolds of the atomic manifold. Supplementing the database manager of the atomic graph with a proper attentive manifold to add meaning to the key-value search is under investigation.

4.5 Algebraizing the Atomic Model on the Tangency Space

The proposed adposition dimensions are compact Lie groups, and they don't define Euclidean charts of the open subsets (U, φ) that when joined smoothly (γ) define a smooth manifold $M = (U, \varphi, \gamma)$. But rather, the proposed dimensions define *various manifolds* that ***maps*** to the atomic manifold on every atomic state (atomic proposition). The adposition dimensions define a local sheaf-theoretic

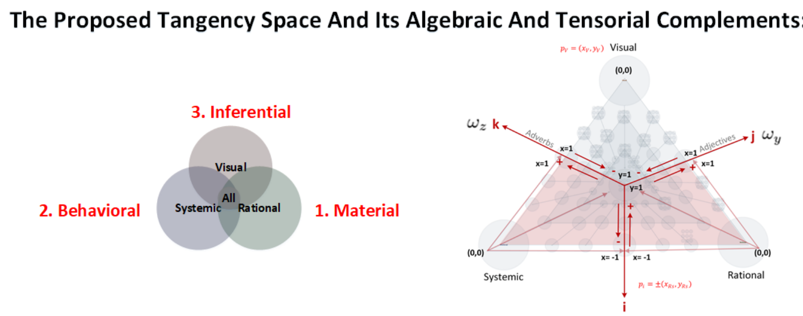

Fig. 10. A proposed Tangency space of the atomic manifold.

differential manifold (M, O_M) (the atomic manifold), where O_M is a sheaf of smooth functions representing the *verb parametrization* from open subsets of M to $\mathbb{R}$, the $1 - form$ differential functions of the speed of change.

Therefore, the compact Lie groups' dimensions may define a Lie algebraic space for the atomic manifold, which is a needed benefit. The paper proposes an *atomic algebraic space*, a Lie algebra-like space, that captures the infinitesimal changes of the material and behavioral perspectives along with the speed of that change as observed by the inferential perspective (see Figure). The atomic algebraic space may be formalized as an identity tangency space of the $if - then$ proposition as the following quaternion-like algebra:

$$\mathbf{Con}_{n \in N} = \pm \frac{\partial}{\partial \varphi_a} < C|| > \mathbf{1} \pm \frac{\partial}{\partial \varphi_b} < C|| > \mathbf{i} \pm \frac{\partial}{\partial \varphi_c} < C|| > \mathbf{j} \pm \frac{\partial}{\partial \varphi_d} < C|| > \mathbf{k}$$

The dimensions $\{\mathbf{1}, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ serve the $if - then$ proposition by assigning the $\mathbf{1}$ as the if condition for the insideness and outsideness $SO(3)$ conclusions. The other three bases $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ represent three perspectival changeability.

5 Discussion

Although the verb parametrization of its arguments may be grounded on how an experiencer observes the speed of changeability between the two state-behavior domains, which may have supportive evidence from the neurological analysis of the well-studied vision system. This related changeability may be valid on the infinitesimal scales, but not on a larger scale. For example, adding conditions to any of the verb's arguments may alter the expected changeability. Meaning that there is a need to address the modeling complexity of sophisticated systems, which is the proposed AtomNN. Doing so generalizes the modeling of verbs into a general-purpose (universal) meaning-based function approximation. Using the AtomNN implies that verbs are functions that may equally model

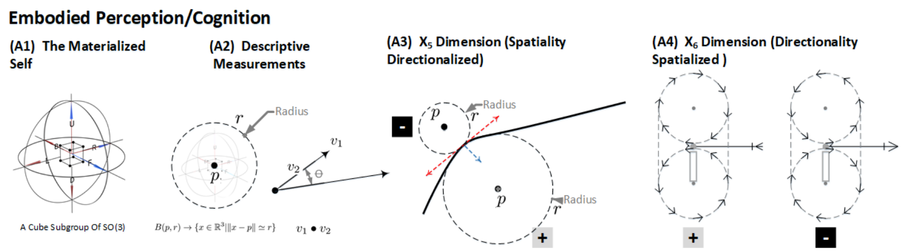

Fig. 11. Exemplar algorithms for extracting material 3D proposition out of spatial experience.

physics, mathematical structures, or any other complex phenomenon of choice, and that is by representing the function parameters as atomic propositions for modeling detailed explainable paths between these arguments over the AtomNN, which comes with gamefiable 3D scenes, the geometric-algebraic simulation by the chamber geometry, white box explainable training process, and the communicative $if - then$ propositions.

The atomic manifold and the atomic graph may be considered as memorized batches of the AtomNNs, which collectively define a growing differentiable atomic graph. Additionally, the capacity to maintain structured memory implies the universal capacity in modeling memory-based biological systems, e.g., the muscular system, community communication, and collective intelligence. For ML, defining an explainable AtomNN that supports recursion, dynamic queryability of different features with different explainable paths of actions may revolutionize the field of DNN and introduce evolutionary ML models to the field. The possibility of using the atomic model with robotics is deliberated in the next section.

Robotics and Conceptualizing the Cognitive System of Agentic Self-Awareness The mathematical structures of $SO(3)$, $SU(2)$ (or quaternions for rotations), and $SE(3)$ (or dual-quaternions for rotations and isometric positions) are used extensively in robotic kinematics, dynamics, or pose modeling. This implies that the atomic concept may merge seamlessly with the robotics best-practices. Nonetheless, the proposed atomic propositions and their memorized experiences over the differential atomic manifold may sustain the self-world entangled relationship modeling. To hold a better grasp of the proposed role atomicity may play in robotics, artificial self-awareness needs to be properly defined.

Definition 7 (artificial self-awareness). *artificial self-awareness outlines agents that manifest the self as an experiential knowledge of any of the following dualistic factors* $f \in \mathcal{F}$*:*

1. *the boy-world duality* (f_0)*: the capacity of defining the body-configuration hierarchy (*$SE(3)$ *hierarchy of the skeletal structure) in relation to the surrounding, e.g., reachability and force production.*
2. *the changing-getting changed duality* (f_1)*: the capacity of changing the world vs. the possibilities of the world to affect the agent's existence, possessions, and goals.*
3. *the punished-rewarded duality* (f_2)*: the capacity to know the requirements and consequences of the rewarding/punishing actions by criticizing the hierarchical structure of the rewarding system's duality.*
4. *the proliferation-protection duality* (f_3)*: the capacity for setting short/intermittent/long-term goals to either achieve and pursue proliferation-related objectives against protecting and avoiding protection-influences.*

All these $f \in \mathcal{F}$ factors of artificial self-awareness may be modeled as

$$describe^{\mapsto},\ (verb^{\circlearrowleft},\ (I^{\mapsto}, O, args_{n\in N}))$$

While $describe^{\mapsto}$ is $f \in \mathcal{F}$. The model queries the extents of these comparabilities, resulting in a proper assessment of the self-world ongoing dialogue. See Figure (11) for embodied spatial propositional population using visual and other sensorial-aided approximations of the seven adposition dimensions.

6 Conclusion

The atomic model is a functional approximation of meaning, according to the paper's theorization. It associates the changeability of the related material and behavioral states with their observed speed of change (the inferential perspective). These infinitesimal changes are algebraically-geometrically simulated on local states of the atomic manifold, to assess context-based similarity, and recorded as hierarchically nested $if-then$ propositions over the atomic graph (long-term memory). The local processes (including the atomic reasoning) are objectively defined, while the atomic propositions are subjectively populated. The populated atomic propositions are experienced or embodied by spatial propositions, behavioral propositions, and inferential propositions, which are materializable as 3D gamifiable scenes. This implies the real-world groundedness of the atomic propositions.

The atomic model may be self-supervised learning using transparent hierarchical backpropagation process that openly learns hierarchical $if-then$ propositions. The same learnability process may be used to train the attentive atomic manifold, which substantiates commonsensical communication (learnt queryability of the atomic propositions). The same approach may be used to train sub-atomic manifolds for specialized cognizing, decision-making, setting goals, planning, and designing tasks.

- hierarchy enhancement: constructing longer, consistent, more detailed paths of the hierarchical $if-then$ may need further algorithmic developments.

- Fourier transforms for the vision system: Developing the vision system ($\equiv$ cognition) that is what you see is what you know, where visual cognition is done by collative bi-directional searching between the sensorial memory and long term memory.
- Natural language generator for the atomic concept: developing a module for parsing atomic propositions into natural language productions to enable the atomic agents to raise queries to LLMs pivots the road for utilizing LLMs as tutors, which is decisive for constructing the evolving atomic agent.
- constructing a controllable, differently-scaled community of robotic agents by structuring the highest proposition in the hierarchical type system and testing the attentive manifold to adhere to the robot's assigned goals.
- studying the hierarchical $if - then$ propositions and their series of algebraic spaces as explainable alternatives for the transformers' COTs.

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