($AtomConcept$) an Atomic Concept for Algebraically, Analogically Auto-Modeling the Witnessed, Symmetric World

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Abstract

The paper proposes the knowledge representation (KR) and its entwined reasoning process of an atomic concept for algebraically, and analogically auto-modeling the formal/semantic manifestations of the symmetric world. The atomic concept is the sole type of the proposed system, and it may be recursively Instantiated to equally model concepts as simple as a word or as complex as the theory of mind agent's self, e.g., the "I", "we", "They", or "It" concepts. The model learns to perform any reasoning task as an analogic learner, knowledge transferability. Algebraic groups are used as the computational core of the atom to homomorphically compare the concepts' structures, and that supports the dynamically evolving analogical compositionality of fundamental concepts into ever more complex ones, which contrasts the top-down, black-box, metric-based-comparability approach of the contemporary field of deep learning, and consequently, it promises a novel trajectory, a shift in paradigm, for the field of artificial psychological intelligence. The paper demonstrates the atom concept as a universal model of language, and the experimentation shows equivalent results with state-of-the-art reasoning tasks. A supplementary work-in-progress local cataloging embedding space for the running the atomic concept over the GPU is proposed.

1. Introduction

The field of deep neural networks (DNNs); as the frontier representative of ML, and in general, the field of AI; proves to be immensely successful in modeling specific-task, nonlinear functions spanning a plethora of applications, e.g., visual/lingual, discriminative/generative models of classification/regression tasks. A DNN model (M) is a differentiable function that may be simply depicted as $M(X, \Theta) \rightarrow \hat{Y}$. The parameters Θ of the predefined model M are optimized to best predict, high-dimensional approximate, the classified/regressed $\hat{Y}$ in relation to the prescribed features X, which is trained by a $(\dot{X}, Y)$ sampled dataset. Nonetheless, the predefined M and its trainable parameters Θ, along with the specifically prescribed features' space X, implies a black box, top-down paradigm with certain inherent limitations. For example, a fair part of the DNN model is trained for the specific-task's optimized representation that is mostly not transferable beyond its particular application, and that is regularly compensated using a sharable high-dimensional Euclidean embedding space. Additionally, DNN models lack the dynamicity needed to adaptively model evolving systems, and although there are plenty of efforts to compensate for these shortages, a model that intrinsically supports knowledge transferability and adaptable dynamicity is needed. The paper closes this gap by proposing a singular standard atomic concept. The atomic concept defines a fundamental unit that may be recursively instantiated to evolve into ever more complex concepts, which is a bottom-up paradigm that is equally capable of modeling concepts as simple as a word and as complex as self-aware agents.

Therefore, rather than prescribing a top-down architecture, it is prescribing an atomic model that dynamically structure to fit the needs of the modeled system is the proposed paradigm. To define such an atomic unit, the paper adopts a tree-perspectival ontology of urbanism (Ezzat, 2019)(Ahmed Ezzat, 2022). A perspectival ontology differs from an existential ontology in that, it is scale-independent. Meaning that it equally maintains its validity on holistic as much as on atomic scales. They are intended to perceive existential ontological objects.

The theory is found to be strongly correlated with natural language. The adopted ontology states that objective reality manifests by the duality between rationality (generalizable characters and properties) and systematicity (transformative interactivity), while subjective visuality (art) is the act of describing this reality. Meaning that properties/characters along with behaviors are objective, e.g., the structural prop-

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erties/characters of any object and their behaviors are unanimously recognizable as such, but describing any/all of these constituents, e.g., useful-useless, good-bad, etc., is a subjective experience, this may not include descriptors (adjectives/adverbs) of physics simulations. It is important to notice that the descriptors are always dual. For example, grasping the descriptor of "possibility" in a given context may not be knowledgeable without finding out what is "impossible" in the same given context.

The proposed bottom-up approach may be compared with the top-down approach based on the following criteria:

- *Similarity Comparison:* the atomic concept binary comparison is Structure-based, and whence concepts are analogically compared. Structure is the faithful representation of the semantic content that the metric-based DNN binary comparisons would lossely measure.
- *Dynamical Evolution:* the model M and the features space X are not predefined, but rather dynamically structured by the reasoning process. Coping with the functional definition of the atomic concept, the function arguments (the features) are dynamically defined and the function itself, as a structure, is dynamically definable. The dynamic definition of the functional representation of the concept and its arguments is what the paper proposes as a dynamical evolution system.
- *Presuming the world's reality:* the world is symmetric. Meaning that for each action there is an anti-action, and for each descriptor there is an opposite polar. Although there may not be a word-to-word symmetry in a given lingual instance, symmetricity is presumed on the conceptual levels, as a concept is much more comprehensive than the mere name (word) that labels the concept.

The following consecutive topics sketch the main facets of the atomic concept and its conceptualizations, prior to structuring the paper. The topics proceed from deliberating the form and semantics of the atomic concept's hypertree KR and its associated reasoning process. After that, the atomic concept is illustrated as a universal model of language, and as a function for discretizing mathematics. Then, the reasoning process is reintroduced as an entwined process with the KR, rather than a separate layer of processing. Lastly, atomicity is deliberated as the fundamental unit that maintains certain formal and semantic units of representation, called *semantic quanta* (Section 2.1) that are presumed to suffice the formal/semantic modeling needs of any system. The section ends by contextualizing the paper with the related works, and then it structures the rest of the paper's following sections.

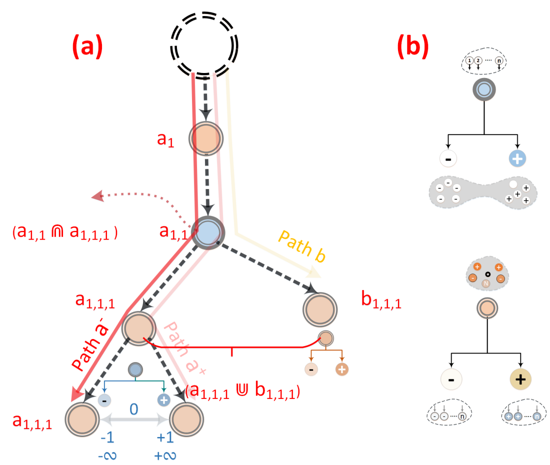

Figure 1. a) the hypertree arranging the two types of nodes over paths. b) polarized descriptors splitting paths after internally/externally structured nodes.

The atomic concept is a knowledge representation (KR) *with* a reasoning process: The knowledge of the atomic concept is represented by a hypertree, which is a generalization of the tree-like graph where the edges are sets that maintain more than just two vertices. The hypertree structures two different types of nodes over paths. Meaning that a concept is just a set of paths, and each of the paths starts by the leading node (Node0), which is the concept's label, e.g., a concept's label of walking, a window, etc. The two dual types of the hypertree nodes are either *internally structured* or *externally structured*. All the paths may either start directly from Node0, or else, share several nodes before proceeding as independent paths, this is called a path splitting. The two node types and the path splitting may be detailed as the following:

1. The *internally-structured* nodes are self-sufficient sets that equally contain actions and their anti-action that counter-act, annihilate, each other. These nodes tend to be accurately modeled mathematically by algebraic Groups, which is the computational core of the atomic concept. Algebraic groups are the foundation for modern abstract mathematics (pure math), theoretical physics, and quantum mechanics. It used in the proposed atomic concept to represent behaviors, such as verbs in natural language.
2. The *externally-structured* nodes don't presume any internal structure, but rather get structured by simply establishing links to other preceding or succeeding nodes. They are suitable for representing generalizable characters and properties.
3. Splitting a *path* into multi-paths imply that earlier

nodes, that start by Node0, are shared between the split paths, and these nodes are shared preconditional nodes. The split may be due to the need for specifying different conditions, e.g., a car with wider doors rather than a car with larger tyres, or due to two opposite descriptive polars, e.g., adjectives or adverbs.

Group-retracted versus group-like processing modes: The hypertree may be processed in two dual modes as the following

Group-Retracted Mode of Processing: the observed/assumed reality cognitively collapses into the potential behaviors associated with it. Behaviors strictly follows the physics conservative laws of energy. Meaning that behaviors come in binary symmetries that can't coexist, in any given moment of observing. For example, a car can either be moving or stopping, turning right or left, but not both at any given time. This symmetric behavior is called a group-retracted mode, that is because behaviors are strictly modeled as algebraic group elements, and the transformable world's states as group actions. Therefore, for group G of collective verbs, the group-based arithmetic operations are strict, as the following:

- Subgroup structure: $H \leqslant G$, then $|H|$ divides $|G|$
- Homomorphism: for the homomorphic map $\varphi : G \to H$, the order of the normal subgroup $ker(\varphi)$ divides $|G|$ and the index of $ker(\varphi)$ in G is $|G : ker(\varphi)| = |H|$
- Group actions: when the group of verbs G acts (transforms) on the states of the world X, the orbit-stabilizer theorem and the Burnside's lemma persist for all $s \in S$ and $g \in G$, the $|G|$, $|Orbit(s)|$, and $|Stabilizer(s)^g|$ are arithmetically enforced by the reasoner according to these laws.

Group-Like Mode of Processing: in the group-like mode, all the world states are in the superposition states, which is the opposite to the group-retracted mode (tightly reasoning about the states in relation to the deformative actions). The only rule preserved is the closure principle of the possible states, e.g., the car can be in superposition of stopping, moving, turning, etc. Bsed on that, this mode may be summarized as:

- All the other algebraic group properties, e.g., associativity, and inverse, are kept loose.
- *Graph-based* operations concerning fragments of relations are collected in this mode, in preparation for elevating them to the group-retracted mode by finding the proper sources on analogy to structure the fragment relations into concept.

The atomic concept is an *entwined* hypertree KR *and* a reasoning process: the reasoning process is responsible for interpreting and generating the form and its related semantic quanta of the hypertree. Algebraic group objects are used by the reasoning process to carry out its mission in either the group-retracted or group like modes. Meaning that there has to be algebraic lineage of symbols to represent of the hypertree. Figure (2) represents the role of verbs in the group retracted mode to structure the group of related verbs $C = c^{-n}.....c^{0}.....c^{+n}$ and their group actions on the verb arguments. Each of the verb elements, e.g., c^{+n}, permutes the states $s_{i \in I}$ for all the I states of $s \in S$. Then $c^{+n}(s_i) \mapsto s_k$, and for all $S = \begin{pmatrix} s_0 & s_1 & \cdots\cdots & s_p \end{pmatrix}$, given that $|S| = p$, $c^{+n}(S) = \begin{pmatrix} s_0 & s_1 & \cdots\cdots & s_p \\ s_a & s_b & \cdots\cdots & s_c \end{pmatrix}$.

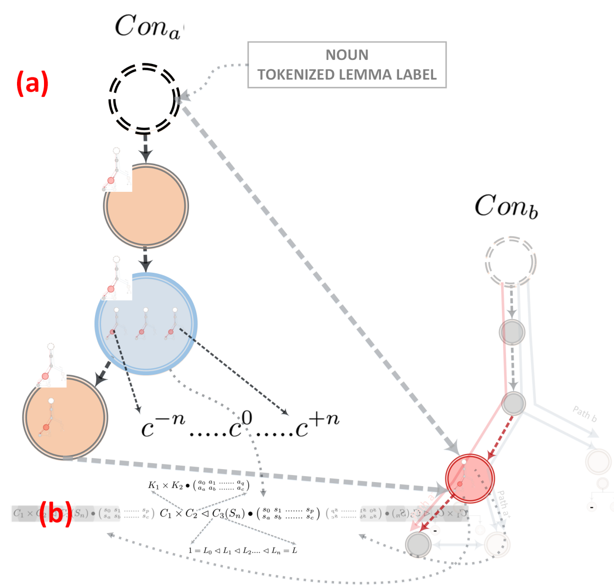

Figure 2. a) When a concept is brought into attention, a node over a path is highlighted. Verbs are algebraic groups of invertable concepts. Con_b is referencing Con_a. b) the graph-extended memorized algebraization of concept recursion

Therefore, verbs collaborate/counter-act to bring transformative changes to the states of the world. For example, the statement of "Sue broke the coconut for Greg with a hammer" implies the state change "Sue" brought to "the coconut" for someone by a certain tool. The significance of the semantic content of such a statement may look to concern Sue, the coconut, or Greg, but in reality such a significance is of no value, who cares for a broken coconut by whom for whom. Other example that may be less materialistic transformations may include "She held the glass", "I dropped/released the ball", "The river froze solid". Although significance of such transformative actions lays on the cognitive infrerentiability as much as on the materialistic transformation. The change the verbs bring is on both the material and the inferentiable states. Meaning that, observing a certain action $c^{+n}(s_i) \mapsto s_k$ implies for the reasoner that other states like $c^{-n}(s_i)$ can't coexist with what is being observed, $Change = MaterialTransformation, Reasoning$. Therefore, the proposed atomic concept suggests that Cognition is the dual to materiality, and it is fundamental, this assumption has plenty of evidence and supported theorizations from the field of quantum mechanics. Achieving this kind of processing is memory-dependent and rests on analogy reasoning. The proposed model respects the symmetricity of behavior in the algebraic representation, and hopes to achieve consistent inferences without any tooling beyond the form/semantics of the proposed atom. This leads us to the next topic of the algebraiziation of the atomic concept.

Algebraizing the atomic concept: Figure (2-b) represents a graph-based, extended algebraic representation. Meaning that the algebraic illustration is momentaneous explication of the atom concept that still holds the memory-based, multi-perspectival development of the atomic concept. Therefore, all the algebraic decomposability, either internal decomposability or its contribution to external decomposability, is maintained in memory. Applying that to the two processing modes imply the following:

- *For the group-retracted mode:* the abstract group classification is maintained in the terms of decompositionality ($\times$ direct products, $\ltimes$ semi-direct products, $\leqslant$ subgroups, $\trianglelefteq$ normal subgroups, subgroup series, and commutativity $ab = ba$), and the related functional mappings and their presumed equalities (homomorphic mapping [map-based equalities], action mapping from g to permuted s [orbit-based class equalities, and map-based equalities])
- *For the group-like mode:* Magma (binary operator closer) is the least quality needed and the other dualities are computed, using analogical structures, before elevating the mode to a group-retracted mode. Such dualities may encompass associative vs. non-associative, commutative vs. non-commutative, identity vs. no-identity, inverse vs. no-inverses, and their decomposable internalized/externalized structures.
- *Finally, for generalization and summarization:* due to the fact that algebraic objects are employed to represent the proposed model, abelianization of algebraic objects, using commutators, or any/all of its memorized decomposabilities consume what the paper presume to be general.

Conceptualizing natural language: parts of speech (POS) and their two open and closed classes are what is meant by natural language. POS unifies all the known instances of the natural language, which are more than seven thousand language instances. The open class (verbs, adjectives/adverbs, and nouns) and the closed class (pronouns, conjunctions, prepositions, and determiner) which may be sufficiently modeled as:

The open classes are faithfully modeled by the hypertree semantics. The verbs have a dedicated node type that is mathematically represented by algebraic groups. Nouns are labels of a hypertree that maintains the generalities vs. specifies or prerequisites vs. implications. The adjectives/adverbs always exist in opposite polars and they split a path into two variations.

The closed classes are evidently modeled by specifying certain hierarchical structures of the hypertree (conjugates and prepositions of manner) or by affecting the way the reasoning process may behave (determiners). The three types of time, place and movement are specifically affected by time, place, and movement prepositions.

Conceptualizing mathematics: math may be split in two continuous and discrete domains. The continuous domain may be modeled as by multi-dimensional linear/nonlinear spaces, while the discrete domain studies structured sets. The continuous domain is modeled by the atomic concept using a bounded continuous space devised by the two descriptive polarized extremes. The path leading to the continuous descriptive domain holds the discrete math possibilities. Meaning that the atomic concept discretizes the adjectival/adverbial continuous domains. The discretization using the structured sets of the node/path arrangement may be explicating in a manner similar to 1st/higher order logic and to category theory.

1.1. Related Works

The atomic concept's contribution may be contextualized with the following related works:

Chains of thought: chains of thought are the state-of-the-art reasoners that broke most of the reasoning benchmarks. They enhance the LLMs reasoning weaknesses using prompt engineering approach to structure the LLMs responses over structured steps. These smaller steps are structured over a path(Wei et al., 2023), a tree(Yao et al., 2023), or a graph(Besta et al., 2023), and they may be further enhanced over recursive, and analogical techniques(Lee & Kim, 2023)(Yasunaga et al., 2023)(Ling et al., 2023)(Jin & Lu, 2023)(Wang et al., 2023)(Zhao et al., 2023). Although, all of these variations are inherent in the proposed conceptualization, the proposed KR/reasoning imply white box, communicative, creative, and theory of mind conceivable applicability, that are, alternatively, inconceivable to similarly maintain using these chains of thought varieties.

Verb aspect/causal modeling: the modeling of verb argument alternations, aspectual, bounded change, causal structure, or both aspectual and causal structure, utilized analogic spaces of time and spatial effects to model the resultant change of verbs' actions(Pinker, 2013)(Pinker & Mehler, 1988). Nonetheless, the proposed atomic concept recognizes change as the dualistic transformation of $\{Materialtransformation, Reasoninginferentiability\}$. Meaning that both of them are the coupled faces of change. Additionally, these approaches have no presumptions about realities of the world.

Homotopy type theory (HoTT): the homotopy type theory is a latest developed version of constructive type theory, which is an alternate of typed λ-calculus (Univalent Foundations Program, 2013) (Sambin & Smith, 1998) (Church, 1940). They study mathematics based on philosophical (logic) and algorithmic bases. Homotopy type theory is a computational core for automated theorem proof assistant. It utilizes groupoids for constructing homotopical definitions of a topological space that represents propositions topologically, and groupoids have close relatedness to what is being offered by the atomic concept's computational core. A groupoid is a partial group, somewhere between a typed-group and a monoid; it mandates the group operators, e.g., $(G, g^{-1}, G \times G \rightharpoonup G)$ to be defined for limited members of G, the closure principle is loose. In a way, the groupoid algebraic type is somewhere between the group-retracted and the group-like types.

GOFAI and expert systems: GOFAI and expert systems are two prominent classical AI paradigms that maintain graph-based knowledge bases of stored triples or if-then rules. These two approaches are deprecated due to their failure to generalize, transfer knowledge, or to manifest continuous domains out of their knowledge bases. Additionally, negating or transforming knowledge segments was a serious obstacle. These challenges are intrinsic constituents of the proposed conceptualization.

1.2. Paper Structure

The paper may be summarized in a nutshell as an atomic unit that structures *hierarchical generalizability* of characteristics and *causal prerequisites* of transformative behaviors. When the model is in a retracted mode, the algebraic group calculations are the sole representation of the conceptualization, . Therefore, section 2 is of high importance, and consequently, the reader may skim it in preparation for Section (3), the reasoning process, and Section (4), the experimentation setup. The appendices extend these notions.

2. The Atomic Concept's Knowledge Representation (KR)

The section sets the abstract mathematical formalization of the hypertree, as a clarified description of its role. The already used jargon may be aligned with the terms specific to the hypertree, which is a variant of a hypergraph(Berge, 1984)(Voloshin, 2009)(Zhou et al., 2006), according to the following: the hypertree is a container of vertices V and hyperedges E $H = \{V, E\}$. The hyperedges $E = \{e_l, \varepsilon_n\}$ are either the node e_l, which is the internally/externally structured nodes, while the other directed hyperedge ε_n are the arcs (segments) connecting the tail e_a to the head e_b of the directed hyperedge, $\varepsilon_n = (e_a, e_b)$. The directed hyper edges structure the already introduced paths of nodes.

The root of the hypertree E_0 is the concept's name, which is labeled by a tokenized lemma. A path $Path_i$ over the hypertree is a sequence of directional hyperedges $Path_{i\in I} = (\varepsilon_{i\in I}^{n\in N})$. Additionally, the polarized descriptors $Desriptor_x(d_x^+, d_x^-)$, e.g., speed(fast, slow), weight (light, heavy), decision (decided, undecided), etc., are labels of the directed hyperedges that split a path into two opposite nodal hyperedges (e_a^+, e_a^-). Therefroe, The compact represent of the path splitting it node e_a is $Path(e_a) \xrightarrow{Descripor_x(d_x^+, d_x^-)} (e_a^+, e_a^-)_x$. It is important to note that the observed world is described by the descriptive polars, e.g., light vs. dark, short vs. tall, etc., that are conceptualized by the proper conceptualizations. This proposes the following representation of the polarized splitting of a path P_x:

$$P_x = \Big\langle \ A_{con_{(a)}} \ \Big| \ [(d_x^+, d_x^-)]_{y\in Y} \ \Big| \ [(e_a^+, e_a^-)_x]_{y\in Y} \ \Big\rangle \tag{1}$$

In case the node of attention is an internally/externally structured node, the representation may be simplified as $\langle Concept \parallel Feature \rangle$.

The Adjacency Matrix of the Atomic Concept's Hypertree Equation (2) represents the adjacency matrix $A_{con_{(a)}}$ of the concept E_0. $A_{con_{(a)}}$ contains the two matrices A_{nodes} and A_{edges} to model the two types of nodal and directed hyperedges, respectively.

The A_{nodes} matrix represents the adjacency matrix for the nodal hyperedges. Each entry in the matrix is either 1,0, depending on whether the vertices belong to the nodal hyperedges or not. On the other hand, the A_{edges} matrix represents the adjacency matrix for the directed hyperedges. Each raw (the directed hyperedge) maintains only two entries, with (-1) to indicate the tail while the (2) represents the head nodal hyperedges. E_0 represents the name of the concept, and each path has to start by an edge that has the nE_0 as the tail node.

$$A_{con_{(a)}} = \begin{array}{c|cccc}
 & E_0 & \multicolumn{3}{c}{\overbrace{e_1 \quad e_2 \cdots\cdots e_l}^{A_{nodes}}} \\
v_1 & \vdots & 0 & 1 \cdots\cdots & 0 \\
v_2 & \vdots & 1 & 1 \cdots\cdots & 1 \\
\vdots & \vdots & \vdots & \vdots \quad \ddots & \vdots \\
v_m & \vdots & 0 & 0 \cdots\cdots & 0 \\
\hline
\varepsilon_1 & 0 & 0 & 2 \cdots\cdots & -1 \\
\varepsilon_2 & 0 & -1 & 0 \cdots\cdots & 2 \\
\vdots & \vdots & \vdots & \vdots \quad \ddots & \vdots \\
\varepsilon_n & -1 & 0 & 2 \cdots\cdots & 0 \\
 & \multicolumn{4}{c}{\underbrace{\hphantom{-1 \quad 0 \quad 2 \cdots\cdots 0}}_{A_{edges}}}
\end{array} \tag{2}$$

The Inducible Equivalent Representations of the Hypertree Group theory unifies algebra, geometry, arithmetic, and analysis under a single umbrella. The paper builds on that and presumes the following multi-domain representation of the hypertree's form and semantics, without an axiomatization that guarantees that.

1st/higher order representation and algebraic representation: the operators $\{\wedge, \vee, \neg_{\uparrow}\}$ are reserved for path-related operations, compositioning nodes over paths. The operands of these operators may be equivalently replaced by algebraic representations. The ordinary operators of $\{\cap, \cup, \neg\}$ are maintained for set-based operations and their quantifications $\{\exists, \forall\}$. The analogic inference crisp operator $\overset{a_i}{\models}: expr \to \{0, 1\}$, that accepts a as a source of analogy, valuates the validity of these expressions.

Continuous polarized descriptor space: the decretive polars define continuous bounded spaces $(-1, 1)$, or $(-\infty, +\infty)$, and these spaces may structure multidimensional spaces. For the sake of simplicity, the paper presumes these spaces as $\{-1, +1\}$

other representations like Category representation, geometric representations (topological and graph-based), and the group representation theory, that manifests over proposed cataloging local embedding space, are all explicable.

2.1. The Semantics of the Representation

The semantics of the formal representations add meaning to the formal units of in-ternally-structured nodes, the externally-structured nodes, the paths linking the nodes, and the preconditions relating different paths, by associating them with the proper semantics. Paths are the building blocks of the atomic concepts and their semantics are dually interpreted by the internally/externally structured nodes. The externally-structured nodes are best-fit to modeling the hierarchical states of the world (detailed generalities), while the internally-structured nodes best-fit to modeling verbs that change these states. The two internally/externally structured nodes dually interpret paths as the following:

- *A single path:* each node on the path is linked to prior nodes (Beforeness), to suc-cessive nodes (Afterness), and they themselves represent the conditions (Whileness) that resides between the shift from "Beforeness" and "Afterness". Based one that, these three along-the-path variations may be interpreted as the following:
  - The externally-structured nodes (states) interpret "Beforeness" as generalities preceding the more detailed "Afterness" under the conditions stated in the node itself "Whileness".
  - The internally-structured nodes (verbs) interpret the path as a chain of causali-ty. They interpret "Beforeness" as the prerequisites needed to precede the be-havior "Whileness" (presuppositions), and such behavior implicates the "After-ness" (implications). Such chain may represent verbs tenses as well, e.g., past/now/future.
- *Multi-paths* share preconditional sets. This means that the "Beforeness" nodes are already presumed or experienced. The splitting is due to different further conditions or due to the descriptive polarized splitting.

The Semantic Units "Semantic Quanta": The introduced semantic interpretations of the hypertree constitute the possibilities of the content any concept may maintain. In other words, no matter what a concept may represent, it may only maintain these generalization, causality, preconditionality content. What differentiates a concept from another depends on a type system, that is described next.

The Type System: Although the semantic quanta presume limited semantic variations, the richness of the semantics of the atom concept rests, as well, on the conditionally (contextually) structured similarities. Types are meant to maintain such context-based semantic-quanta. For example, the agent-world existential ontology may imply certain types like space, time, and transformative actions.

Analogical Knowledge Transferability: Whatever an instantiated concept may represent, e.g., the "I" concept, a mountain, a wall, etc., there are certain semantic content that may be analogically transferred from one concept to another. Analogical learning implies selecting a proper reference prior to translating its content to the novel concept. Both of these operations require context-based similarity/oppositionality comparability, which are tightly related to the constructed type system.

2.2. The Computability of the Atomic Concept

The appendices explicate a work-in-progress specially devised manifold to embed the atomic concept in. This local cataloging embedding space is shaped by the three perspectives and enables the atomic concept to run over the GPU for parallel processing. The proposed embedding space id the finite field F that the group verbal elements $g_{i \in I}$ acts on its points $v_{k \in K}$ by $g_i(v_k) \mapsto SL_2(F)$, and whence applying the robust group representation theory on the atomic concept.

3. The Atomic Concept's Reasoning Process

The mission of the reasoning process is to analogically interpret/produce the atomic concept's knowledge representation, the hypertree. In fact, the reasoning process is the only tool that may semantically and mathematically interact with the hypertree. For doing so, the reasoner may equivalently recognize the semantics and the formali-ties of the hypertree, and that is done over two layers of abstraction. The higher se-mantic layer deals with the semantic units of the KR, while the lower layer utilizes algebraic groups as the computational core that mathematically processes the KR as a hypertree/hypergraph. Finally, the reasoning is algorithmically explicated as a pro-cess that may be activated over a series of queries/responds in relation to a thinking process or over a cross-agents established conversation.

3.1. The Semantic Reasoner (High-Level Reasoner)

The high-level reasoner interprets the semantics of the formal representations. This implies that the formal units of internally-structured nodes, the externally-structured nodes, the paths linking the nodes, and the preconditionals relating different paths, are all associated with the proposer semantics. These semantic quanta are processed over a query-response dialogue to activate an inference system that utilized an analogical comparer, as follows:

The Query vs. Respond Duality This mechanism unifies all the thinking process, aligning conceptions, and finally, aligns the two semantics/formal (computational) layers. This suggested mechanism may not only be considered as a convenient tool of analysis, but rather with neuro-logical, philosophical supported evidence. The query-response duality occur over the duals of similar objects and their related contexts (see Figure 3-b).

The Inference System Each semantic quantum implies certain inferable implications. These inferences are the implications of asserting(observing)/assuming(presuming) any of these semantic quanta. These inferable implications may be summarized as the following:

- *The context is known:* if either Beforeness, Afterness, or both are known then con-clude similarities may be reasoned about.
- *The similarities are known:* if similarities are known, then there must be a context that either caused such similarity, may be caused by it, or both.

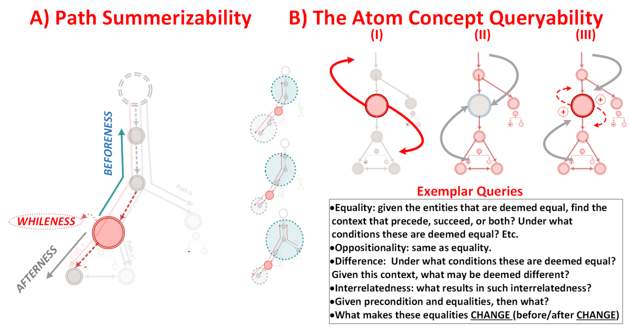

Figure 3. a) beforeness, afterness path summerization over its nodes. b) the query-response mechanism to align reasoning layers

The Evaluating Reasoner (the Comparer) The basic discrete process the reasoner conducts is to analogically compare concepts for the best similarity. This id followed by evaluating the graph-based operations, e.g., update, insert, create, and delete, to transfer the proper knowledge from the source to the target.

3.2. The Formal Reasoner (Low-Level Reasoner)

Verbs, or behaviors in general, change the states of the world. The states of the are world are described by the externally-structured nodes that are linked over paths. Such interrelatedness between the internally-structured nodes (verbs) and the exter-nally-structured ones (states) would entail the following cognitive and mathematical implications:

- *cognitively speaking*, retracting noticing the potential actions associated with the current states, for planning, preparing the states for certain actions, etc. retracting the graph as behaviors.
- *mathematically speaking*, the hypertree as a graph, as a topology, as a dynamical-ly formalized algebra, as an algebraic group, the mathematical and computational core. Concluded Logical (1st-higher order) language.

4. Experimentation Setup

The hypertree is a data structure that is utilized by the reasoning process. The paper includes, as well, a work in progress for running the atomic concept on the GPU by manifesting a cataloging local embedding space, as an application of the representation theory of the group structures. Based on that, the repository dedicated to the experimentation, which is accessible at https://github.com/Anonymous200024/AtomConcept_V0.000, is structured into a CPU and GPU folders. The CPU version maintains the data structures needed for the hypertree, e.g., the baseNode, the Externally-StructuredNode, the Internally-StructuredNode, the Edge, the Path, and finally, the AtomicConcept that lists all the populated paths. All these data types maintain references to algebraic group instances, which is implemented using the open source library of SymPy (Meurer et al., 2017). Neo4j is used as the graph database manager for the MemoryManager package (Neo4j, 2012). The Reasoner package is developed as depicted in the paper. For the GPU version, a deep convolution neural network is trained for assigning the proper concepts on the proper contextual location in the embedding space, see the appendices for further details.

The proposed KR is developmental. It develops over three phases. This developmental nature is easier to grasp through Equation (2). In Equation (2), the A_{nodes} matrix builds on the set of vertices $\{v_m\}$. This set is the fundamental set of concepts needed for the nodal hyperedges $\{e_l\}$ to be defined, which in turn are required for the set of directed hyperedges $\{\varepsilon_n\}$ to be defined. Although all of these sets need to be structured differently for any concept they represent, the set of $\{v_m\}$ is fixed, they are the fundamental words of natural language or the basic terms of any domain-specific knowledge.

Consequently, the first phase is the initialization phase that initializes the set of vertices $\{v_m\}$. It is important to observe that any of the fundamental vertices $\{v_m\}$ is merely a concept, which in turn is structured by a hypergraph similar to Equation (2) that structures the other fundamental vertices as part of that concept. The second phase is the documenting phase.

It is important to keep in mind that the paper is of theoretical nature, and whence, the experimentation is meant to demonstrate what is being theoretically laid throughout the paper in a computationally sensible manner, not to irrelevantly attest to performance superiority. The two developmental phases, along with the local embedding space function, may be described as the following:

1. *Initializing the fundamental concepts:* Popular KG databases are queried for each word found in the language. The prominent databases of WordNet, ConceptNet, FrameNet, and VerbNet, construct KGs of lingual words linked with labeled relations(Miller, 1995)(Speer et al., 2016)(Ruppenhofer et al., 2005)(Schuler, 2006). The words and their opposites are queries with the relations acting as the conditional nodes. Statements defining these named relations are parsed and natively represented by the hypertree concepts.

2. *Conceptual compositionality (the documenting phase):* In this phase, the agent grows its ability in composing concepts and in raising/answering questions, using query-able active self-learning techniques. The queries are concerned with the features and their classified concepts. The learner queries LLM machines, like ChatGPT and GPT-4, for finding answers, which are parsed and encoded by a conditional-binary hypertree(OpenAI, 2023).

3. *Constructing the embedding space:* see the appendices for the experimentation related to constructing the work-in-progress local embedding space using a convolutional neural network trained function to spatially embed contextualized semantics.

4.1. Results

The proposed model shares plenty of traits with the human agents, that may be summarized as the following:

- It is developmental. Meaning that, it develops over stages prior to maturity. Therefore, the model memorizes its experiences that are monolithically modeled by the proposed methods.

- It is recursive, and such reasoning mechanic is evident in human thinking. Additionally, the neurological anatomy of the brain suggests the homogenous unit that structure the brain, the way atomicity intuits.

- It is analogical, and that is a grounded explanation of the zero-shot learning capacity of the human agents, aka., knowledge transferability.

- It is a communicative white box. Meaning that the observed visual describers are conceptualized by selecting the proper sources of analogy and then presume structural similarities of the observed describes, this is called understanding. Additionally, the model performs its reasoning over query-response paradigm, and that may enable it to query databases, e.g., LLMs and the other mentioned knowledge bases, for answers.

These are the ground the proposed model presumes its superiority's upon. To demonstrate that, several models that propose the geometric structuring of knowledge, similar to the proposed model, are selected (see Section 1.1 for further details). The main two differences between the atom concept and these models are:

1. The atom concept memorizes its experiences, Neo4j is the graph database manager used for that goal, while the selected COTs variants are prompt engineering approaches of LLMs.
2. The atom concept learns by building parsers and evaluators of these parsers, a mission that can be manually populated and analogically executed by the built conceptualization to cover all the possible/expected outcome of the modeled task.

The following table complements the detailed analysis accessible on the code repository at https://github.com/Anonymous200024/AtomConcept_V0.000.

Table 1. Qualitative comparison between the atom concept and variants of chains of thought over common sensual and arithmetic reasoning benchmarks.

SCHEME	GSM8K	SVAMP	CSQA
CHAIN-OF-THOUGHT (COT)	×	√	×
TREE-OF-THOUGHT (TOT)	-	-	-
GRAPH OF THOUGHTS (GOT)	-	-	-
ATOM CONCEPT	√	×	√

5. Conclusion

The paper theorizes the atomic concept that is constituted by the hypertree representation, the reasoning mechanic, and the local embedding space, all of which were defined and harmonized due to the underlying three-perspectival ontology of urbanism. This neuron maintains the seed of intelligence, the creativity of cognizing, and its dynamically agglomerated neural networks, which would in turn further agglomerate toward the creation of evolutional knowledge representation that may equally sustain perceptual and higher cognitive processes. The atomic concept is found to be inseparable from language, on both levels of representation and reasoning. For example, the fundamental concepts may be paralleled with lingual words, and their compositions may be paralleled with matching lingual variants like sentences, paragraphs, etc. This pivots the path for a novel open-box distributed collaborative intelligence.

Impact Statements

This paper presents work whose goal is to manifest a fundamental atom concept to structure novel bottom-up conceptualization based on pre-encoded belief about the world. the machine is believed to be creative, analogical reasoning, and maintain human-like thinking traits. There are many potential societal consequences of our work, none which we feel must be specifically highlighted here.

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The appendices include three sections concerning proposed framework may be parallel computationally performed over the GPU using the propped local embedding space, the way natural language may be modeled by the proposed atomic concept, and finally, an illustration of how a creative design process may be established aided by the atomic concept (conceptualizing spatial design creativity).

A. Matrix-Based Computations of the Proposed Framework

A.1. Defining the Embedding Space

The proposed hypertree is a variant of a hypergraph that has been deliberated, so far, for its efficiency in representing concepts. Nonetheless, for the reasoning mechanics to achieve its goal of responding to queries with versions of justified answers, it has to employ certain operations over the hypertree that may be computationally impractical(Knauer, 2011)(Voloshin, 2002)(Cooley & of Birmingham, 2009)(Kepner & Gilbert, 2011). Meaning that what has been laid out so far could be considered a problem definition rather than a wholistic optimized practical algorithm to process concepts. The limitations of depending solely on the hypergraph represntations may be summarized in the following three factors that justify the need for embedding the hypergraph as an alternative complementary form of represntation:

a. Grounded communication: communication is mostly reflective, or an instantaneous agreement on which features of which concepts are intuited in the given discourse. Such groundedness manifests over repeated communications with the rest of the communicating population. Searching the hypertree for an instantaneous response is less responsive.
b. Graph match-ability: hypergraph isomorphic or holomorphic matching has no known algorithm. The same may be considered for conditional-binary hypertrees(Knauer, 2011)(Voloshin, 2002)(Kepner & Gilbert, 2011)(Berge, 1984).
c. GPU optimized parallel processing: partitioning and coloring hypergraphs, whicg are needed for representation or for parallel processing, are NP-hard problems. Additionally, the proposed reasoning mechanics would highly benefit from parallel algebraic calculations(Zhou et al., 2006)(Shi & Malik, 2000)(Buluc et al., 2015).

Based on these factors, the paper devises a novel semantic space for embedding the induced hypertree. The spatial and semantic qualities of the proposed local space are shaped by the three perspectives, and they are introduced in the first subsection. Following that, the possibilities of embedding the hypertree representation in the proposed local embedding space, and then to the algebraic practice of the reasoning process over the embedded hypertree are discussed.

Defining spatiality of the proposed embedding space: Embedding a mathematical structure (A) in another (B) implies that certain characteristics of the embedding space (B) are meant to preserve certain selected characters of (A). The contentious opposition between the three perspectives shapes the proposed local embedding space. Meaning that the spatial coordinate system and its associated semantics are defined by the contrast between the three perspectives. Any spatial point in the embedding space should be belonging to any of the three perspectives' characteristics much more than it would for the others. This paves a differentiable space (see Figure (3)[A]), which devises the discrete coordinate system of its points P as $P = \{p_i \in (D_{Rational}, D_{systemic}, D_{visual}) \mid D \in (x, y)\}$. All the D dimensions have two $x \in (-1, 1)$ and $y \in (0, 1)$ coordinates that are discretized by,$\{i, n, k, l, m\} \in Z$. This constructs the proposed coordinate system of the embedding points P as:

$$P = \{p_i \in (D_{Rational}, D_{systemic}, D_{visual}) \mid D \in (x, y), \{i, n, k, l, m\} \in Z\,, \\ x \in (-n/k, n/k) \mid^{k}_{(n=1)}, y \in (0, m/l) \mid^{l}_{(m=1)}, i \in (0, 2kl)\} \tag{3}$$

Therefore, each point in the embedding space may be represented as $p_i = ((x_r, y_r), (x_s, y_s), (x_v, y_v))$. Knowing that the rational and systemic subspaces are opposites entails that $p_i = (\pm(x_{rs}, y_{rs}), (x_v, y_v))$. Due to the fact that the visual perspective has its own artistic concepts that would have an independent spatial resolution from the rational/emotional perspectives, we may end up with the $p_i = \pm(x_{rs}, y_{rs})$,as the positive location is rational while the opposing identical opposite point is negative and the visual perspective as $p_V = (x_v, y_v)$.

Standardizing the local embedding space semantics, embedding the hypertree representation, and performing the reasoning mechanics over the space: The purpose of the local embedding space is to facilitate the reasoning process's operations, most challengingly traversing the hypertree in search for equalities (isomorphic/homomorphic comparisons), which is not computationally viable. The hypertree is the truthful representation of the concept. The hypertree truthfully represents similarities/oppositions under the given hierarchical contextuality (hierarchical conditions). Noting that similarities

may encompass those of the conditional sets as much as those of the classified tree-leaf sets along with their hypertree-devised structures. Embedding the hypertree should truthfully represent the *contextuality* and *structurality* of the original representation as much as possible.
The paper proposes assigning semantic contents to all the spatial points in P. Meaning that for all $p_i \in P$ the function $\varepsilon(p_i) \longrightarrow \langle Concept_i \parallel \emptyset \rangle$ is defined. In other words, a $Concept_i$ is selected and anchored at each p_i to catalog similar/opposite concepts. The *contextuality* is achieved by restating the assignment function as $\varepsilon(p_i) \longrightarrow \langle Concept_i \parallel Feature_{m \in M} \rangle$, which means that the features of the anchored concept $Concept_i$ would spread over parallel $|M|$ multi-local embedding spaces. The function $\varepsilon_{embed}(p_i) \vdash (\langle Concept \parallel Feature_{m \in M} \rangle_{i \in I} \times \langle Concept_{j \in J} \parallel Feature_{k \in K} \rangle \longrightarrow T_{\mathbb{R}}$ measures the best point from the $2ml$ points (the embedding resolution) over the $|M|$ multi-local spaces for embedding $\langle Concept_{j \in J} \parallel Feature_{k \in K} \rangle$. $\varepsilon_{embed}(p_i)$ measures which $m_{q \in M}$ best fit the embedding $Featture_{k \in K}$ depending on the contextuality of its prior conditional sets, this is why the codomain of $\varepsilon(p_i)_{embed}$ is a real number tensor $T_{\mathbb{R}}$. The function ε leads to the embedding function $E_{\varepsilon}(\langle Concept_{j \in J} \parallel Featture_{k \in K} \rangle) \longrightarrow (p_{i \in I})_{m \in M}$, which present the local space's version of *contextuality*. Additionally, the local *structurality* of the embedded $\langle Concept_{j \in J} \parallel Featture_{k \in K} \rangle$ is defined as a transformative operator $\odot$, which is regularly a matrix that transforms from the embedding dimension to the tree-based local coordinates and its dimensionality, e.g., $(2x2, 3x3, ..etc.)$, is based on the length of the hypertree paths it transforms. This leads to the final form of the embedding function:

$$E_{\varepsilon}(\langle Concept_{j \in J} \parallel Featture_{k \in K} \rangle) \longrightarrow ((p_{i \in I})_{m \in M}, (\odot_{j \in J})_{k \in K}) \tag{4}$$

Consequently, the embedding space is an adaptive catalog that promises to align massive concepts over the available $2ml \times |M|$ embedding resolution. The embedding works in synchrony with the hypertree, and the degree of the hypertree-dependence measures the truthfulness of representation. The anchored concepts $Concept_i$ may be selected based on different criteria, such selection may change over time based on the feedback. The semantics of $\varepsilon_{embed}(p_i)$ is a convolution DNN, and the training dataset is composed of thousands of augmented data out of several manually populated concepts. . The proposed local embedding highlights the advantage of human-like semantic-based matching over the mere morphological-based (formal syntactic) alternatives(Dai & Yeung, 2006)(Arockiaraj et al., 2015)(Wu, 1985)(Sundara Rajan et al., 2015)(Abraham et al., 2007).

A.2. Mtrix Operations

One of the purposes of the proposed local embedding space is to embed the hyper graph using the equation of $E_{\varepsilon}(\langle Concept_{j \in J} \parallel Featture_{k \in K} \rangle) \longrightarrow$
$((p_{i \in I})_{m \in M}, (\odot_{j \in J})_{k \in K})$.

Doing so resolves plenty of complexities related to the hypergraph traversals for matching and for homomorphic operations. The two outputs of the equations are either the spatial location of the hypertree concepts that may be defined by Equation (1):

$$E_{\varepsilon}(\langle Concept_{j \in J} \parallel Featture_{k \in K} \rangle) \longrightarrow (x_j, y_j, 1) \tag{5}$$

On the other hand, the group representation by matrices is independent of the embedding space. meaning that the matrices

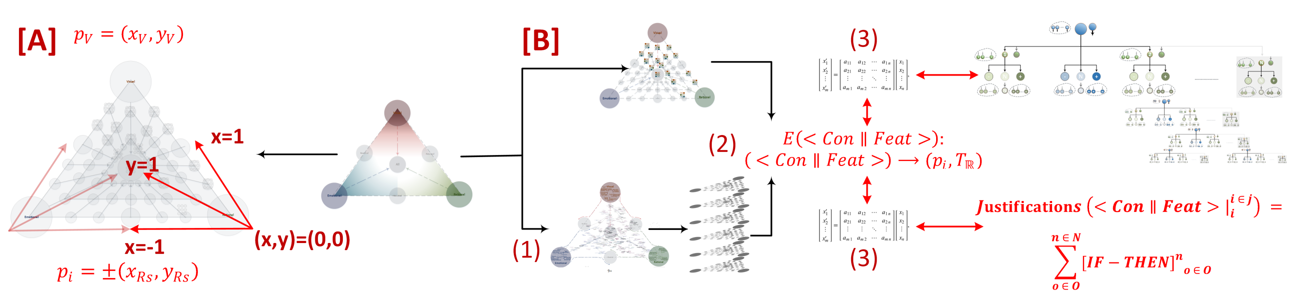

Figure 4. From left to right, [A] the spatial coordinate system. [B] (1) the training dataset for the semantic embedding function defined in (2). (3) the hypertree/reasoning transformative matrices

only materialize the verbs' group structures. Consequently, to use the groups' matrix representation, there is a need for a transformation between the embedding space coordinate system and the matrices bases used to represent the verbal groups. The matrix $T_{Con(a)}$, in Equation (2), is a good example of how the group's matrix representation may by mapped by $T_{Con(a)}$ to translate the embedding $(x_j, y_j, 1)$ coordinate.

$$T_{Con(a)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t(x) & t(y) & 1 \end{bmatrix} \tag{6}$$

$$R_x = \left\langle \ A_{con_{(a)}} \ \middle| \ [(d_x^+, d_x^-)]_{y \in Y} \ \middle| \ [(e_a^+, e_a^-)_x]_{y \in Y} \ \right\rangle \tag{7}$$

As for the operator $\odot_{j \in J})_{k \in K}$, Equation (3) represent the content that this operator should be used to embed the structure of the hypertree in the local embedding space. Mapping matrices similar to $T_{Con(a)}$, in Equation (2), may be used for the three component representing any path R_x, as in Equation (3).

A.3. GPU-based Experimentation Setup

Constructing the embedding space: The local embedding semantics depends on the function $\varepsilon_{embed}(p_i)$, which measures semantic similarities between the concept $Concept_i$ that is anchored to all $p_i = \pm(x_{rs}, y_{rs}) \in P$, and the embedding concepts $Concept_(j \in J)$. There are $m \in M$ embedding spaces, and each has a spatial p_i , each of these m spaces has a concept used by $\varepsilon_{embed}(p_i)$ to define contextual semantics, and that is how the proposed embedding act as a catalog of massive embedded concepts. The challenge is that $\varepsilon_{embed}(p_i)$ is semantic-based, and the three perspectives, although contrasting, are abstract vague entities. Consequently, human assistance is required to define $\varepsilon_{embed}(p_i)$. This is done by manually populating eight concepts to reflect the harmonious conceptual space resulting from the three perspectival contentions. The concepts are then augmented into thousands of sample points using Glove (the words' vector representer)(Schuler, 2006). The training dataset is used to train a DNN fomred by convolution/pooling represntation layers that are followed by a feedforward classifier, which suffices the $\varepsilon_{embed}(p_i)$ semantics classification purposes.

B. Conceptualizing Natural Language

Language is the most sophisticated production of human agents that clearly distanc-es them form the other conscious live beings. But, what is language? For this paper, parts of speech (POS) and their classes are what is meant by natural language. POS unifies all the known instances of the natural language, which are more than seven thousand language instances. Meaning that any language instance maintains two classes of POS; the first is the open class (verbs, adjectives/adverbs, and nouns), while the second is the closed class (pronouns, conjunctions, prepositions, and deter-miner). The open classes are subject to constant additions/alterations, while the closed classes may barely change [9]. The atomic concept is a content-based parser, verbs are central to its modeling, and it seems sharply tailored to fit the representation needs of natural language by modeling the two word classes as the following :

- The open classes are faithfully modeled by the hypertree semantics (see Figure 5-B). The verbs have a dedicated node type that is mathematically represented by algebraic groups. Nouns are labels of a hypertree that maintains the generalities vs. specifies or prerequisites vs. implications. The adjectives/adverbs always exist in opposite polars and they split a path into two variations.

- The closed classes are evidently modeled by specifying certain hierarchical struc-tures of the hypertree (conjugates and prepositions of manner) or by affecting the way the reasoning process may behave (determiners). The three types of time, place and movement are specifically affected by time, place, and movement prepositions (see Figure 5- C and D).

B.1. Representing Verbs by the Atomic Concept

Verbs are central to language, and the paper believes that the proposed retractability of the world states into their potentially associated behaviors is what causing this importance. The subjects, objects, and spatial/temporal modifiers may all be consid-ered arguments to a verb. These arguments are all listed before and after a verb node with their states prior and after the transformative action of the verbs. This simple semantic arrangement may not only represent tenses (series of verb nodes), but also hold the capacity of modeling any verb with all the valences, e.g., intransitive, transi-tive, ditransitive and double transitive verbs. Additionally, all the argumentative modeling challenges, e.g., aspectual, bounded change, causal structure, or both aspec-tual and causal structure, comes natural to the proposed atomic concept.

B.2. Lingual Parsing

The semantic quanta presumed by the atomic concept are believed to suffice the semantic modeling needs of any domain of knowledge, including natural language. Nonetheless, there is a need for a parser to translate the formal representations of these domains, e.g., lingual syntax, topological images/objects, or sound waves, into the atomic KR representation. Doing so interrelates perception to cognition by the memorized database of conceptions, and such interlinking is strongly supported by modern neurology. These parsers are themselves concepts and they be manually constructed or semantically self-taught by learning agents.

C. Conceptualizing Spatiality

Space is the element that unifies the built environments' seemingly disparate fields of architecture, urbanism, and internal design. Space is the conceptual element that humans, as creative cognitive agents, may interact, evolve, express their dreams, function, and maintain societal behaviors inside/around that element. Nonetheless, space has fundamental elements, and these elements, to a satisfactory degree, are explicated by the paper's adopted three-perspectival ontology of urbanism. Entwin-ing these fundamental elements, the concept of "It", with the theory of mind's de-vised concepts of "I", "We", and "They" psychologically constructs what may be known as space (see Figure 6). Neither the spatial elements nor the attributions of the theory of mind is comprehensible without the aid of language. The way they may be conceptualized and the how that would aid a creative-based design process are expli-cated in sequence.

C.1. Structural Fundamentality

Rem Koolhaas has a significate contribution in explicating the basic elements of architecture. Nonetheless, the three perspectival ontology comes equipped with the fundamental elements of spatiality [12], and whence it acts as a perspectival ontology as much as existential ontology for the urban environment (see Figure 6). These fundamental spatial elements span the activities, functionality (protection vs. demarcation), and materiality along with their features.

C.2. Form = Semantics (Conceptualizing Spatial Fundamntality)

A main benefit of the fundamentality proposed by the atomic concept is that form and semantics may be equally conceptual-ized. Doing so, makes the material or the semantic content lingually explicable, and consequently, the degree of cognition, reinterpretation, or rephrasing are guaranteed to be creatively explicated.

C.3. The Theory of Mind

Any human agent may manifest over the existential duality of proliferation vs. safe-ty, and to a certain degree, that shapes the concept of "I", with all its perception, cognition, simulated feelings and actions that may affect or get affected by urban constructs (spatiality). The concept of "I" may define, and get redefined by, the concept of "Us", against the concept of "Them". These First person view vs. third person views that interactively interactive with the spatial fundamental elements of urban constructs may set a novel attainable guideline of how to consider spatiality.

C.4. The designer's supported creativity

The designer's enriched awareness about the spatial elements' conceptualizations, form the theory of mind of the different category of intended users/experiencers and their interrelated perspectives, would imply computationally-aided creativity-enriched design process. This would imply a designer being a philosopher in greater capacity to query/respond to what may have been, otherwise, impractically time-consuming facet of the design process.

C.5. Spatial Cognition Change Rates (the Role of Music in Design)

It is worth to mention that spatial cognition may change over time, e.g., the way "I" may affect or get affected by the environment "It". Nonetheless, the way such mu-tual definition may change over time may signify some musical effect of such change, e.g., rhythm, tempo, dynamics, harmony, etc. This may suggest that music is an innate in the cognitive system and would, consequently, require special attention from the designer.

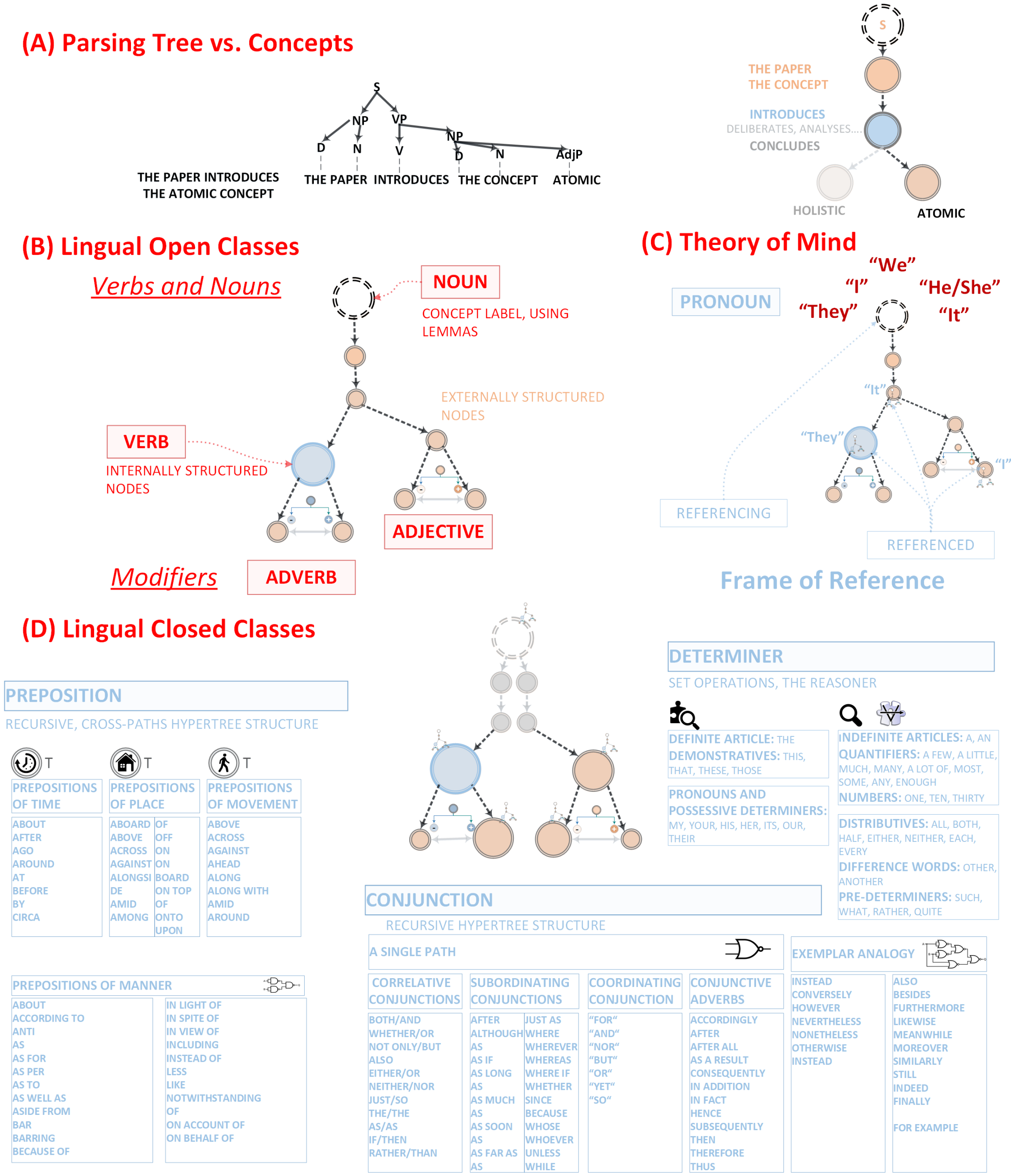

Figure 5. The capacity of the atomic concept in semantically modeling natural language.

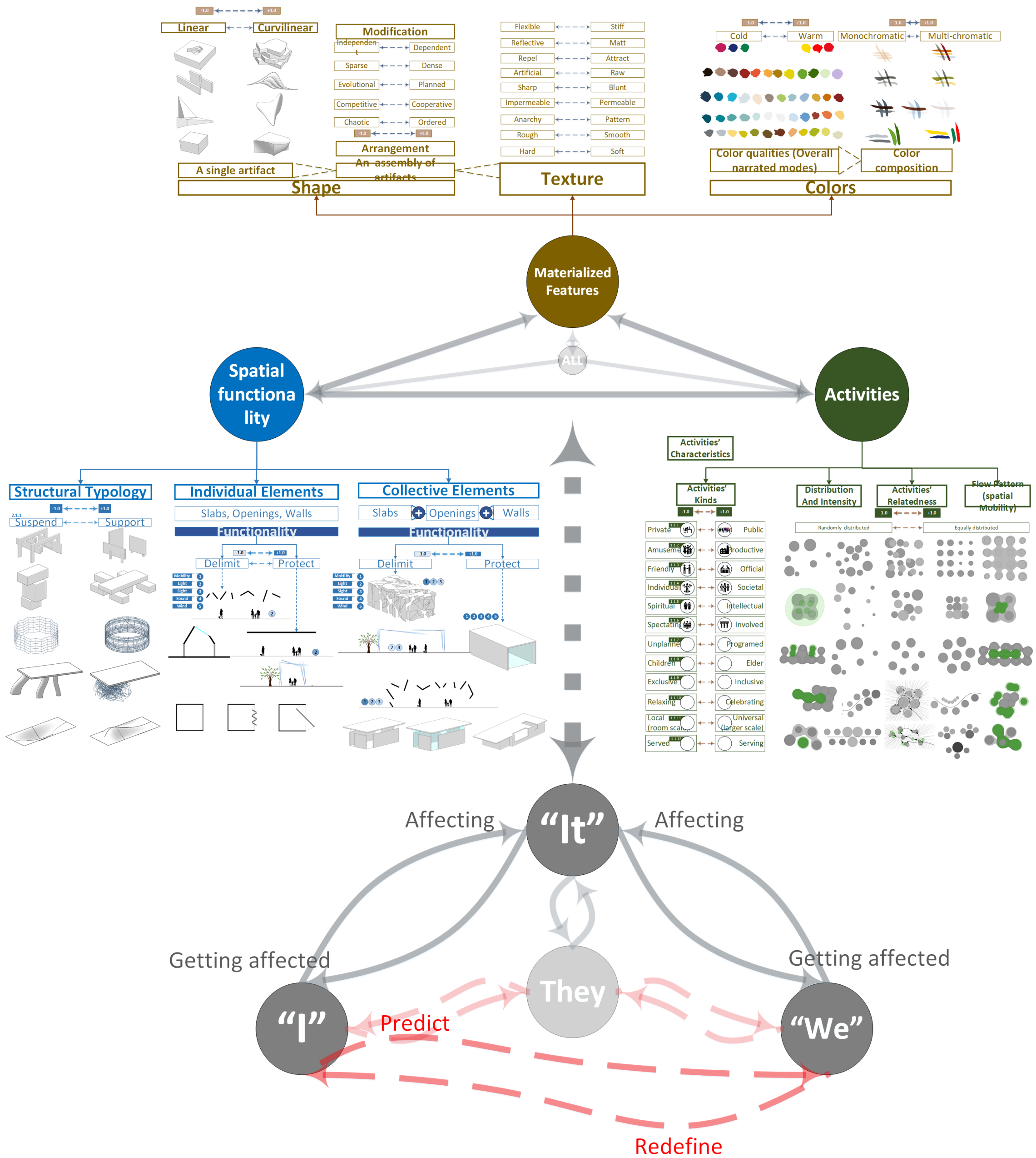

Figure 6. The conceptualization of space by the fundamental elements proposed by the three perspectives and the "I", "We", and "They" concepts interrelated to the environment's "It" Concept.

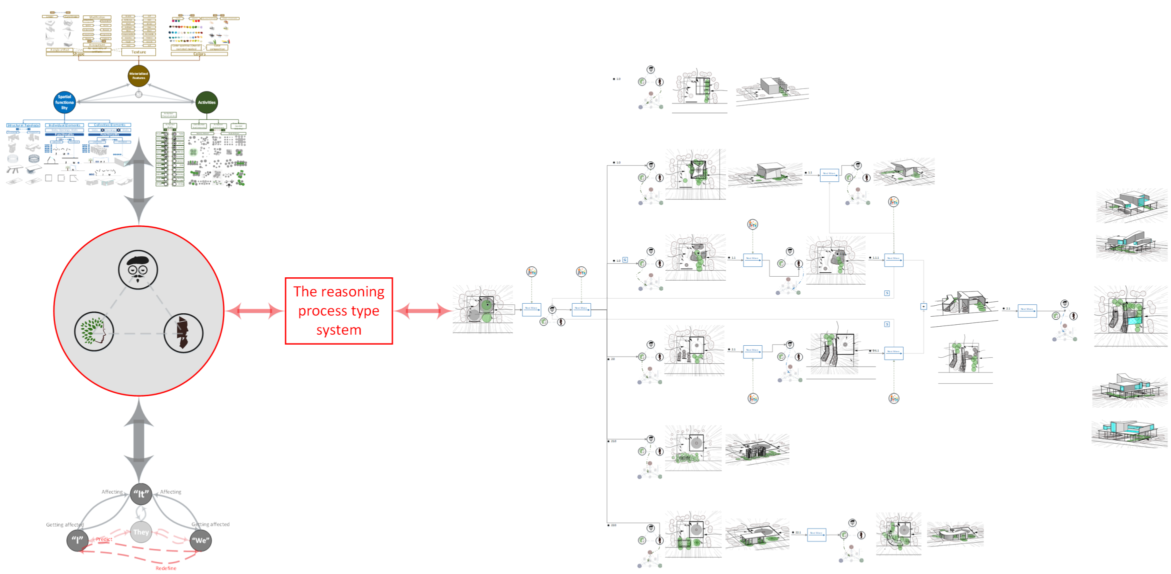

Figure 7. A user-space informed design decisions for automating designer-aided creative design process.